AN INVESTIGATION OF THE DAMPING INFLUENCES ON THE TORSIONAL OSCILLATION OF AN ENGINE CRANKSHAFT

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SYMBOLS

a_n = Holzer table amplitude at n^{th} station.

b,c = damping factor per unit vibrational velocity

c_r = radial clearance.

d = diameter.

e = eccentricity of journal centre.

f = eccentricity ratio of transverse vibration.

g = gravitational constant.

h = hysteresis loss per unit volume.
or
oil film thickness.

i = number of engine cylinders.

j = $\sqrt{-1}$

k_n = torsional stiffness of n^{th} shaft.

l = length of connecting rod.

m = generalized mass or exponent in the expression for hysteretic damping.

n = order number of vibration or number of cycles.

p = damping "loss factor" (Draminsky formula)

q = flow of lubricant per unit width.

r = radius, of bearing or journal.

t = time.

A = empirical constant in Ker Wilson's expression for engine damping.
or
factor in the expression for hysteretic damping.

E = Young's modulus.

F = force or generalized force or rate of dissipation of energy in Shannon formula.

G = modulus of rigidity.

I_n = mass moment of inertia at n^{th} station.

I, J = second moment of area.

K = stiffness

L = bearing length

M = dynamic magnifier or generalized mass.

Q_n = n^{th} order harmonic component of torque.

R = crank radius.

T = generalized torque.

U = surface velocity.

W = transverse load on journal, or strain energy.

ΔW = energy dissipated per cycle.

δ = the logarithmic decrement.

ε = eccentricity ratio $\frac{e}{c_r}$

ζ = ratio of damping to critical damping.

η = absolute viscosity.

θ_n = amplitude of vibration at n^{th} station.

θ_{o_n} = equilibrium amplitude at n^{th} station.

λ = eigenvalue.

τ = shear stress.

ψ = attitude angle of journal.

ω = angular velocity of rotation of shaft rad/sec.

ω_f = forcing frequency

ω_n = natural frequency.

1. SUMMARY

A 6-cylinder Ford Zephyr engine was used with a variable speed external drive to determine the effect of various engine parameters on the damping of torsional oscillation of the engine crankshaft.

At resonance when the amplitude of oscillation was approximately $\pm 2^{\circ}$ and the induced stress in the shafting ± 8 tonf/in^2 the contributions of various sources to the damping were found to be of the following order:-

material or hysteretic	6% to 8%
bearings, shearing of oil film	8% to 10%
bearings, pumping oil round the bearing annulus	80% to 85%
piston rings	2% to 3%

The damping coefficients for the two major sources, hysteretic and oil pumping have been calculated and used in matrix calculations on an Elliot 803B computer to demonstrate how this data, when obtained, may be used as an alternative to the overall engine magnifier system now in common use in industry.

The introduction examines very briefly the current methods used in industry, to predict the overall dynamic magnification for a new engine design. It is in this application that the need exists for better methods of determining the

resultant amplitude of oscillation.

Shannon (reference 4) has already shown that deflections of the crankshaft resulting in oil pumping are responsible for much of the damping. A new approach used in this report using velocity damping coefficient has been compared with Shannon's energy approach and gives good agreement on the amount of deflection of the crankshaft needed to give the damping loss.

The work on piston ring damping in this report which shows the damping due to piston rings to be only of the order of 3% is, as far as is known, new.

2. INTRODUCTION

The methods of calculation used to determine the response of reciprocating engine installations to torsional oscillation were well developed before the advent of digital computers. These methods have continued in use in industry without change though the calculation is now always done by digital computer. It is proposed in this investigation to examine the sources of damping in an engine installation and to find out if these can be employed in a form of calculation better suited to the computer.

The practical method of calculating the response of a reciprocating engine installation to torsional oscillation of the crankshaft system has been for many years as follows:-

(a) Idealise the crankshaft and its attached shafts and masses into an equivalent torsional system of discrete rigid masses connected by weightless elements of torsional stiffness. Use the Holzer table or other method to determine the natural frequencies and the associated normal modes of the undamped system.

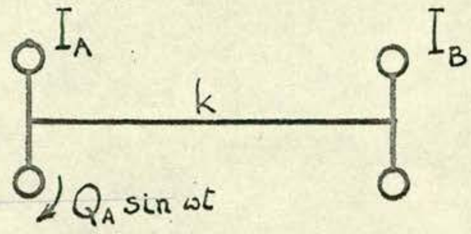
(b) Obtain the harmonic coefficients of the disturbing torques applied to the system. These are usually only those due to gas forces and inertia which are then combined vectorially. As the input energy from each cylinder crank to the n^{th} order vibration is proportional to the product of the combined harmonic component Qn of the above torques and the amplitude of vibration at that crank the resultant input of energy for all m cylinders is proportional to $\sum_1^m Q_n a_n$, or $Q_n \sum_1^m a_n$ as the individual cylinder lines are usually indentical, where $\sum_1^m a_n$ is the vector summation of the relative amplitudes of vibration at the cylinders, the phasing being that of the Q_n. A value of $\sum a_n$ can be obtained from the Holzer tables for each mode of a particular shaft system and for different firing orders. This is a valuable means of determining the order numbers of the torque which will cause major excitation.

(c) The idealised system of (a) is then treated as an even simpler two mass system and an 'equilibrium' deflection obtained for the above applied torque. This corresponds to the response of the system as the forcing frequency approaches zero. The 'equilibrium' amplitude when multiplied by a 'dynamic magnifier' or 'magnification factor' gives the amplitude response of the system. The evaluation of the equilibrium amplitude and dynamic magnifier will be described below.

The earliest explanation of the equilibrium amplitude concept of a multi-degree of freedom system was probably given by Dr. J. Lockwood Taylor in Engineering, May 29th 1931.

Equilibrium Amplitude.

The mathematical model of the torsional system is reduced to



and the equations of motion are:-

$$I_A \ddot{\alpha}_A + k(\alpha_A - \alpha_B) = Q_A \sin \omega t$$
$$I_B \ddot{\alpha}_B + k(\alpha_B - \alpha_A) = 0$$

where α is the angle of oscillation.

For the undamped sytem

let $\alpha_A = \theta_A \sin \omega t$

then $$\theta_A = \frac{Q_A(k - I_B\omega^2)}{\omega^2(I_A I_B \omega^2 - k(I_A + I_B))} \quad \ldots \quad \ldots \quad \ldots (1)$$

The natural vibrations which can be determined from the homogeneous equations can be ignored because in any practical installation they will be quickly damped out. They lead to

$\omega_n^2 = 0$ or to $\omega_n^2 = \frac{k(I_A + I_B)}{I_A I_B}$ i.e. to "rolling" of the system or to vibration at the natural frequency.

Consider now a slightly different approach to the same problem. If the external vibratory torque $Q_A \sin \omega t$ is suddenly removed the system will be left vibrating in a one-node mode at the natural frequency.

Energy imparted to the system = $\frac{1}{2} Q_A \theta_{OA}$

The maximum kinetic energy = $\frac{1}{2} \omega_n^2 \left(I_A \theta_{OA}^2 + I_B \theta_{OB}^2 \right)$

hence $$\theta_{OA} = \frac{Q_A}{\omega_n^2 \left\{ I_A + \frac{I_A^2}{I_B} \right\}}$$

If $I_B \to \infty$ the system becomes a simple torsional pendulum and $\theta_{OA} \to \frac{Q_A}{I_A \omega_n^2} = \frac{Q_A}{k}$

which is the 'static' or 'equilibrium' deflection of the torsional pendulum.

Equilibrium amplitude for a multi-mass system.

θ_{01} = equilibrium amplitude at a selected datum (usually No.1 mass) where the relative amplitude for the normal mode under consideration is a_1 (usually normalised to 1 on the Holzer table).

θ_{on} = equilibrium amplitude at the n^{th} mass where the relative amplitude for the mode under consideration is a_n derived from the Holzer table).

then $$\theta_{on} = \theta_{01} \frac{a_n}{a_1}$$

and $$\frac{1}{2} \sum Q_n \theta_{on} = \frac{1}{2} \omega^2 \sum I_n \theta_{on}^2$$

$$\theta_{01} \sum Q_n \frac{a_n}{a_1} = \omega^2 \theta_{01}^2 \sum I_n \frac{a_n^2}{a_1^2}$$

$$\theta_{01} = \frac{\sum Q_n a_n}{\omega^2 \sum I_n a_n^2} \qquad \dots \quad \dots \quad \dots (2)$$

Equilibrium Stress

Equilibrium stress at any point in a system is defined as the maximum cyclic stress at that point while the system oscillates at the equilibrium amplitude.

$$\tau_{on} = \text{equilibrium stress at } n^{th} \text{ shaft}$$

$$= \theta_{01} \frac{r \sum I \omega^2 a}{\frac{\pi}{32} d^4} \qquad \dots \quad \dots \quad \dots (3)$$

where r = minimum radius of the shaft at the n^{th} shaft in the system = $\frac{d}{2}$

$\sum_{1}^{n} I \omega^2 a$ = inertia torque from Holzer table at n^{th} shaft in the system.

Magnification factor(or dynamic magnifier).

The magnification factor M is defined as

$$M = \frac{\text{vibration amplitude}}{\text{equilibrium amplitude}} = \frac{\theta_{VA}}{\theta_{OA}}$$

$$\theta_{VA} = M\,\theta_{OA}$$

$$= \left(\frac{\omega_n^2}{\omega_n^2 - \omega^2}\right)\left(\frac{Q_A I_B}{I_A(I_A + I_B)\omega_n^2}\right) \dots \quad \dots \quad \dots (4)$$

assuming damping to be light, and $\omega \neq \omega_n$

But the assemby is capable of 'rolling' under the externally applied torque with both masses in phase.

$$\theta_R = -\frac{Q_A}{\omega^2(I_A + I_B)}$$

Complete forced vibration at A

$$\theta_{FA} = \theta_{VA} + \theta_R$$

$$= \frac{(k - I_B\omega^2)\,Q_A}{\omega^2\left[I_A I_B \omega^2 - k(I_A + I_B)\right]}$$

which is the same result as was obtained directly from the equations of motion (Eqn.No.1.)

In the above, magnification factor was used in the form $\frac{\omega_n^2}{\omega_n^2 - \omega^2}$ which is nearly correct only when damping is low. A more complete expression is

$$M = \frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \frac{c^2}{k^2}\omega^2}}$$

when c is a damping torque per unit vibratory velocity.

When the system is not in resonance it is customary to use the magnification factor in the form $\frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$ and this appears to be satisfactory, but when the system is resonating it is essential to have a value for the damping, which alone limits the amplitude of the oscillation. The effect of the amount of damping present on the natural frequency and on the phase relationship between the masses is considered negligible.

The methods which are in use in industry to determine the amplitude of vibration during design will now be examined in some detail. They all purport to give an overall value of M for an engine but they are mostly based on statistical evidence of tests on particular types of engine and are usually only accurate for that type of engine which the originator studied.

Methods of Predicting the Dynamic Magnifier

Shannon's method, reference 4.

Shannon's was one of the earliest attempts to determine the sources of damping in an engine installation. Tests were conducted on a small 4-cylinder engine with only two main bearings, one of which was a roller bearing. It was therefore not representative of the type of engine for which the damping data is most often required.

Shannon demonstrates that

$$\theta_1 = \frac{\sum Q_n a_n}{c\,\omega_n \sum a_n^2}$$

$$\theta_{01} = \frac{\sum Q_n a_n}{\omega_n^2 \sum I a_n^2}$$

where θ_1 = vibration amplitude at No.1.

θ_{01} = equilibrium amplitude at No.1.

c = damping factor per cylinder per unit vibrational velocity.

I = moment of inertia per line, average if necessary, then $\sum I a_n^2 = I \sum a_n^2$

$$M = \frac{\theta_1}{\theta_{01}} = \frac{I\omega_n}{c} = \frac{I\omega_n^2\,\theta_1 \sum a_n^2}{\sum Q_n a_n} \quad \ldots \quad \ldots \quad \ldots (5)$$

Shannon believed that hysteretic damping was negligible and ignored it completely. He further demonstrated that viscous drag torque in the main bearings responding to speed fluctuations caused by the vibration could account for only 7% of the vibrational energy loss in his engine. Damping in his opinion was therefore due not to simple shear resistance but to movement of the journal in the flooded bearing.

An expression was derived for the rate of dissipation of energy per unit width of a journal in an oil filled bearing.

$$F = \frac{\eta f^2 \omega_n^2 \pi^3 d^3}{10\, c_r} \qquad \dots \quad \dots \quad \dots (6)$$

Part of Shannon's work was a study of the results of torsiograph tests on a number of engines from which he observed that the dynamic magnifier seemed to be a function of the normal mode. Engines with a single node remote from the crankshaft had the lowest dynamic magnifier and those with a node near the centre of the crankshaft the highest magnifier.

An expression was produced for a dynamic magnifier which is dependent solely on the shape of the elastic curve (normal mode).

$$M = 58 - 48 \frac{\Sigma |a|}{i} \qquad \dots \quad \dots \quad \dots (7)$$

Ker Wilson's method, reference 2.

It is assumed that the energy dissipated by damping can be expressed by :-

$\Delta W = A \omega^m \theta^n$ where A, m, n are constants determined by the type of damping.

In a multi-mass system this becomes

$$\Delta W = \omega^m \theta^n \Sigma(A a^n)$$

The input energy per cycle is

$$\delta W = \pi \theta \Sigma(Q a)$$

At resonance these are equal

$$\therefore \quad \theta_r^{n-1} = \frac{\pi \Sigma(Q a)}{\omega^m \Sigma(A a^n)}$$ where θ_r = amplitude at reference mass at resonance

but $M = \dfrac{\theta_r}{\theta_o}$

and $\theta_o = \dfrac{\Sigma Q a}{\omega^2 \Sigma(I a^2)}$

hence $M^{n-1} = \dfrac{\pi \omega^{2-m} \Sigma(I a^2)}{\theta_o^{n-2} \Sigma(A a^n)}$

If the damping were linear or viscous $m = 1$ and $n = 2$ but Ker Wilson states that in fact the damping is independent of frequency and proportional to the cube of the amplitude,

therefore $M = \dfrac{H}{\sqrt{\theta_o}}$ where H is a constant for the particular system and the mode of vibration

or $M = \dfrac{h}{\sqrt{\tau_o}}$ where τ_o is the maximumequilibrium stress, and h is another empirical constant derived from tests.

The results of measurements made on a large number of engines are plotted and found to lie within a band given by the following formula

$$M = \frac{A}{\sqrt{\tau_o + 16}} \quad \text{where A is between 350 and 650} \qquad \ldots \quad \ldots \quad \ldots (8)$$

A mean value of 500 is suggested with a tendency for medium speed engines (say up to 600 rev/min) to have dynamic magnifiers in the upper region of the scatter band while high speed engines appear to be biased to the lower region.

Draminsky's method, reference 6.

Dr. Draminsky attempted to obtain an explanation of the damping in a single cylinder engine of 8.66 inches cylinder diameter driven by external power. He decided that the main sources of damping were hysteretic and oil damping and gave simple expressions for calculating these for a single cylinder engine. These formulae were further developed for use in the design of multi-cylinder engines.

He also uses the concept of equilibrium amplitude and a dynamic magnifier but uses the inverse of M where

$\frac{1}{M} = P$ called by him the 'reduced relative damping' term. (It is more usually called the loss factor and denoted by η but Draminsky's term and symbol will be used in the following examination of his methods).

He defines equilibrium amplitude in the usual way as

$\theta_o = \frac{\sum Q_n a_n}{\omega_n^2 \sum I_n a_n^2}$ but calls it the 'reduced statical deflection'.

He then defines the 'reduced relative damping' term as

$$P = \frac{\sum c_n a_n^2}{\omega_n \sum I_n a_n^2}$$

where C_n = the damping coefficient at each point n in the system, per unit vibrational velocity, and the actual forced vibration amplitude is then $\theta = \frac{\theta_o}{P}$

The derivation of the expression for p is :-

ΔW = work absorbed per cycle for damping

$= \sum \pi c_n \omega_n a_n^2$

and W = total maximum kinetic energy of the vibration

$$= \sum \tfrac{1}{2} I_n a_n^2 \omega_n^2$$

$$\text{then } p = \frac{1}{M} \doteq \frac{\Delta W}{2\pi W} = \frac{\sum c_n a_n^2}{\omega_n \sum I_n a_n^2} \quad \ldots \quad \ldots \quad \ldots (9)$$

Using this type of analysis he derives expressions

for p_h = hysteretic 'reduced relative damping coefficient'

and p_b = 'reduced relative damping coefficient' for oil damping.

Taking p_h first:-

ΔW = work absorbed by hysteresis

$= \int h.d\,v.$ where h = hysteresis loss per unit volume of the material and depends on the shear stress.

h usually takes the form $h = A\tau^2$

W = total maximum strain energy

$$= \int \frac{\tau^2}{2G}\,dv$$

$$\text{then} \quad p_h \doteq \frac{\Delta W}{2\pi W} = \frac{G}{\pi}\frac{\int h\,dv}{\int \tau^2 dv}$$

If $h = A\tau^2$ then $p_h = \dfrac{GA}{\pi}$

Experimentally, for normal crankshaft steel on his size of engine (subject to stress not exceeding the fatigue limit) he shows:-

h = hysteretic loss per cu.in. of shaft material

$$= 0{\cdot}004\,\frac{\pi\tau^2}{G}$$

$$= 1{\cdot}05 \times 10^{-9}\,\tau^2 \text{ in.lbf.in}^{-3} \quad \ldots \quad \ldots \quad \ldots (10)$$

A very simple analysis of the oil resistance to eccentric movement produces an expression for damping at the bearings.

W_R = equivalent rotating mass at crankpin radius.

W_p = equivalent reciprocating mass.

n = order number of the oscillation (no. of vibrations per revolution).

α = retardation angle between the centre line of the crank and the line of eccentricity, shown experimentally in Draminsky's rig to be 15° consistently.

The expression for total damping at the main bearings in a multi-cylinder engine becomes:-

$$P_b = \sum \frac{W_R R^2 a_n^2}{g \sum I_n a_n^2} \cdot \frac{c_r n}{R \sin\alpha} \quad \ldots \quad \ldots \quad \ldots (11)$$

W_R would become the vector sum of half the unbalanced masses of the two adjacent cranks.

Sweeping assumptions were then made about some of the parameters and of the ratios between others.

An average value for $\frac{W_R R^2}{g \cdot I}$ for 'normal' engines was taken as 0.85 and the similar expression involving the reciprocating mass was taken as 0.40 leading to:-

$$P_b = (0.85 + 0.40) \frac{c_r n.}{0.25 R} \quad \ldots \quad \ldots \quad \ldots (12)$$

For multi-cylinder engines assuming a node at the centre of the crankshaft and taking the masses from each adjacent crank at a mean phasing of 120° this expression became :-

$$p_b = 3.3 \frac{c_r n}{R}$$

$$\text{or} = 2.5 \frac{c_r n}{R} \quad \text{if counterweights were present.}$$

$$= 0.1\,n \quad \text{assuming a mean value of 0.04 for } \frac{c_r}{R}$$

Draminsky advocated a constant 0.6 per cent damping for hysteresis and foundation damping and gave the formula for an engine with a node at the centre and up to 10 cylinders (marine mostly) as

$$P_{total} = (0.1n + 0.6) \quad \text{per cent.}$$

For engines with a flywheel and therefore a node near one end, bearing pressures due to twisting became more important than those due to accelerating forces and the formula was modified to

$$P_{total} = \left[(0.004\,i^2 + 0.06)n + 0.6\right] \text{ per cent} \quad \ldots (13)$$

where i = number of cylinders.

Den Hartog's Method.

After putting forward a formula for dynamic magnification in earlier editions, in the fourth edition of his "Mechanical Vibrations", p.207, Den Hartog returns to statistical evidence that the magnifier varies between 25 and 100, usually being 50! No other methods are offered. Under this system he comments that the observed amplitude will never be more than twice or less than half the 'calculated' value which is as good as any of the empirically derived multipliers will achieve.

Comment on the empirical methods.

Not all the known methods have been considered here but enough has been given to show their nature. The final comment might seem to be Den Hartog's latest but this is not good enough. Designers frequently want to know the effect of changes of parameters such as oil viscosity, bearing clearance, number of piston rings, piston ring wall pressure, stress levels in driven shafting, etc., and a method which allows these changes to be taken into consideration when the existing magnification factor is already known would be valuable. In addition, with the increased use of computers more elegant means of solution of the frequency problem than the Holzer table are possible and it would be very desirable to be able to include damping coefficients in the equations of motion and to obtain the total response of the system.

Scope of this project.

Matrix methods seem to offer the best method of application to the computer of the torsional vibration calculations and it is proposed to examine these.

The assessment of damping coefficients is a necessary step before matrix methods can be applied to the damped equations of motion and a rig has been designed to enable some of the damping factors in a particular engine to be evaluated. An attempt will be made to generalize the findings.

In particular it seemed at the start that three sources of damping would be of greatest importance:-

(1) hysteretic damping or internal friction of stressed shafting,

(2) oil damping at bearings,

(3) friction damping at pistons and rings.

Various researchers have made different statements about these. Ker Wilson (2) in his earlier editions assumed only hysteretic damping, Shannon (4) assumed only oil damping at bearings. Statements have been made that (3) is a negligible effect but no proof or demonstration has ever been given.

3. THE RIG

3.1 Photograph No.1 shows the complete installation which was designed for this project.

A Ford Zephyr 6-cylinder petrol engine was modified by having the cylinder head, the flywheel, the chain drive to the camshaft and all other driven parts including the internal oil pump removed. A separately driven oil pump with a heater in the oil circuit enabled oil pressure and temperature to be controlled.

An external drive shaft, Fig.2 called the intermediate shaft was fitted between the engine crankshaft and a large flywheel, Fig.4. Five intermediate shafts of different diameter were provided so that changes in the natural frequency and in the level of shear stress could be effected. The intermediate shafts were made of the same nominal 0.2% C mild steel and normalized at about $800^{\circ}C$, but were not all made at the same time and differences in their internal damping is apparent.

The other elements of the elastic system were the crankshaft fig.3 and the pistons and connecting rods. The essentials of the torsional system are shown in fig.5 .

A 15 h.p. electric motor and variable speed drive (Pye TASC unit) provided means of rotating the crankshaft at any speed up to 3,000 rev/min. A splined clutch (photographs nos. 3 and 4) was provided between flywheel and chain drive so that the driving mechanism could be completely disconnected from the engine torsional system at speed to enable the damping of the engine only to be observed. Comparison of results showed that the effect of the drive on the amplitude of the engine torsional oscillation was not measurable.

The oil used throughout all these tests was Mobiloil Special for which the temperature-viscosity relationship is shown on fig. 19

Instrumentation of the Rig

A Southern Instruments inductive torsional vibration transducer type G.218 was fitted to the free end of the crank-shaft to record effectively the amplitude of torsional oscillation at No.1 cylinder line. An electro-magnetic transducer type G.308 was fitted adjacent to the coupling to record the arrival of No.1 piston at top dead centre. The arrangement of these two transducers is shown in photograph No.2 The main transducer G.218 is intended for use with a frequency modulated system. Changes in its inductance sensing element frequency modulates a signal from the oscillator type M.700. The signal is subsequently pre-amplified in an M.R.200 F.M.unit and demodulated before going to the ultra-violet recorder as a fluctuating voltage. The full equipment, see photograph No.1, as provided, can be statically calibrated by a micrometer which deflects the seismic core of the transducer. The static calibration of both of the G.218 transducers used during the tests was checked on a dynamic test rig made up for the purpose, see photograph No.5. The response was found to be better than the manufacturers' claim and is shown for one of the transducers in fig. 10 .

Typical records from the U.V. recorder are shown in figs. No.6 to 9.

Fig. No.6 is a record of 2nd order oscillation taken at the peak of resonance when fitted with the 0.60 intermediate shaft and No.1 piston only with 3 piston rings.

No.7 is the 'run-down' record, as above, when the clutch was withdrawn at a speed just above the 2nd order critical speed.

No.8 is a calibration record.

No.9 is the typical log decrement curve obtained for the intermediate shafts.

The method of obtaining the decrement curves is shown in photographs 6, 7, 8 and 9.

3.2 Frequency Calculations

Crankshaft (fig No.3)

Moments of inertia

Wt. of piston and gudgeon pin	1 lbf $3\frac{1}{4}$ oz	
" " 3 piston rings	$1\frac{1}{2}$ oz	
	1 lbf $4\frac{3}{4}$ oz =	1.297 lbf
" " small end of con. rod with bearing		0.313 lbf
Total equivalent reciprocating weight		1.610 lbf
Wt. of large end of con.rod with bearing		1.156 lbf

Equivalent revolving weight $= 1.156 + \frac{1.610}{2} = 1.956$ lbf

Equivalent revolving $WR^2 = 1.956 \times 1.565^2 = 4.79$ lbf in^2.

Nos. 1, 2, 5 and 6 lines.

Crankpin	2.68 lbf in^2	
$\frac{1}{2}$ second web	13.25 " "	
Small web	10.52 " "	
$\frac{1}{2}$ journal	0.60 " "	
	27.05 " "	
less hole	.91 " "	
	26.14 " "	
piston and con rod	4.79 " "	
	30.93 " "	= 0.08 lb in sec^2

Nos. 3 and 4 lines.

Crankpin	2.68 " "	
outer web	10.52 " "	
$\frac{1}{2}$ inner web	8.89 " "	
$\frac{1}{2}$ journal	.60 " "	
	22.69 " "	
less hole	.91 " "	
	21.78 " "	
piston and con rod	4.79 " "	
	26.57 " "	= 0.0688 lb in sec^2
Flywheel		3.06 lb in sec^2

Torsional Stiffness of the Crankshaft.

The method used is given in detail on p.56 of ref.1.

L_e = length of 'equivalent' circular bar of diameter 2.377" (the journal diameter)

K = torsional stiffness lbf.in./rad.

	L_e in.	K Lbf. in./rad.
Free end keyway to No.1 line	32.516	1.09 X 10^6
No.1 line to No.2 line	2.963	8.95 "
No.2 line to No.3 line	6.315	5.63 "
No.3 line to No.4 line	3.828	9.29 "
No.4 line to No.5 line	6.315	5.63 "
No.5 line to No.6 line	2.963	8.95 "
No.6 line to flywheel coupling	5.021	7.07 "

Torsional Stiffness of the Intermediate Shafts.

The purpose of the intermediate shaft (Fig.2) was to tune the system to a suitable natural frequency of torsional oscillation so that the 3rd order vibration could be studied, and also take the maximum shear stress in the system in such a way that the hysteretic damping could be assessed. Five were made of different diameters so that changes of natural frequency and of stress level could be studied.

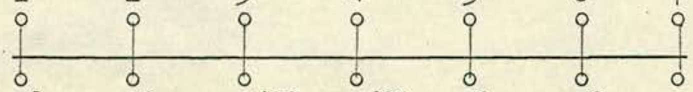
TABLE 1

Shaft No.	Diameter in. Nominal	Actual	Torsional Stiffness Lbf.in./rad.	Natural frequency of rig (6 pistons) C.P.M.	3rd order engine speed r.p.m.
1	.60	(.604)	22600	2270	756
2	.65	(.652)	31200	2665	889
3	.70	(.702)	41800	3081	1028
4	.75	(.750)	55000	3530	1177
5	.80	(.802)	71500	4018	1340

Calculation of natural frequency of torsional oscillation.

The natural frequencies and normal modes have been calculated both by Holzer tabulation and by matrix methods (the latter to be described later) in the Elliot 803 digital computer from programs written in Algol 60 .

When fitted with all six pistons and connecting rods the mathematical model of the system used was:-

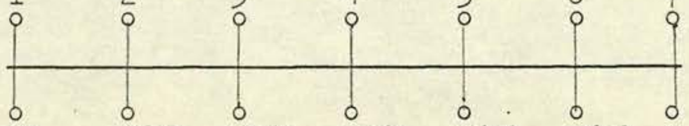


No.	1	2	3	4	5	6	7	
M.	.08	.08	.0688	.0688	.08	.08	3.06	lb.in sec^2

K. 8.95 5.63 9.29 5.63 8.95 * X 10^6 lbf.in./rad.

* for stiffness of the intermediate shaft see table 1, page 22.

When fitted with only the piston and connecting rod of No.1 line the masses became :-



No.	1	2	3	4	5	6	7	
M	.08	.0678	.0563	.0563	.0678	.0678	3.06	lb.in.sec^2

the stiffnesses remaining as before.

The print outs of natural frequency and modal shapes are given in appendix 1.

3.3 Calculation of the vibratory torques.

The inertia torque due to acceleration of the reciprocating mass in each cylinder line is described by a series :-

$$T = Q_1 \sin \phi + Q_2 \sin 2\phi + Q_3 \sin 3\phi + \text{-----}$$

where Q_n the n^{th} order component of the inertia torque

$$= \frac{W}{g} \omega_f^2 R H_n$$

where W = weight of the reciprocating parts

ω_f = angular velocity of rotation of crank rad/sec.

R = crank radius

Hn = a coefficient depending on the ratio $\ell/R = n$

ℓ = connecting rod length.

$$H_1 = \frac{1}{4n} + \frac{1}{16n^3} + \frac{15}{512n^5} + ---$$

$$H_2 = -\frac{1}{2} - \frac{1}{32n^4} - \frac{1}{32n^6} + ---$$

$$H_3 = -\frac{3}{4n} - \frac{9}{32n^3} - \frac{81}{512n^5} ----$$ (reference 1, page 276)

In the rig $n = \frac{5.3125}{1.565} = 3.4$

$\therefore H_2 = -0.500$

$H_3 = -0.227$

Rig with 6 pistons, 3rd order vibratory torque, and equilibrium amplitude.

TABLE 2

Intermediate shaft dia.in.	ω_n^* (expt) rad/sec.	$\omega_f = \frac{\omega_n}{3}$ rad./sec.	Q_3 lbf.in.	$\sum Q_3 a$	$\pm\theta_o$ rad.	$\pm\theta_o$ deg.	$\pm\tau_o$ lbf/in^2
0.60	238	79.3	14.6	87.3	.00294	.168	1782
0.65	279	93.	20.0	119.8	.00294	.168	1930
0.70	311	100.4	23.4	139.2	.00294	.168	2065
0.75	361	120.5	33.6	200.0	.00294	.168	2190
0.80	395	131.7	40.2	248.5	.00294	.168	2340

*where it differs from the calculated ω_n the observed value has been substituted in the calculation.

Rig with No.1 piston and connecting rod only.

2nd order harmonic torque and equilibrium amplitude

$$\frac{W}{g} R\overset{2}{H}_2 = \frac{1.61}{386} 1.565^2 \times 0.500 = .0051 \text{ lbf.in.}$$

TABLE 3

Intermediate shaft dia. in.	ω_f^2	Q_2 lbf.in.	$\sum Q_2 a$ *	$\pm \theta_o$ rad.	$\pm \theta_o$ deg.	$\pm \tau_o$ lbf/in^2
0.60	16,380	83.4	83.4	.00324	.1853	1930
0.65	21,600	110.0	110.0	.00324	.1853	2090
0.70	27,900	142.4	142.0	.00324	.1853	2240
0.75	37,800	193.0	193.0	.00324	.1853	2380
0.80	44,500	227.0	227.0	.00324	.1853	2540

*These values of torque are subject to a correction for the inertia couple effect of the connecting rod, see Appendix 3.

3rd order harmonic torque and equilibrium amplitude

$$\frac{W}{g} R\overset{2}{H}_3 = \frac{1.61}{386} 1.565 \times 0.227 = 0.00232 \text{ lbf.in.}$$

TABLE 4

Intermediate shaft	ω_f^2	Q_3 lbf.in	$\sum Q_a a$	$\pm \theta_o$ rad.	$\pm \theta_o$ deg.	$\pm \tau_o$ lbf/in^2
0.60	7,520	17.44	17.44	.000657	.0377	393
0.65	10,000	23.2	23.2	.000657	.0377	424
0.70	12,850	29.8	29.8	.000657	.0377	454
0.75	16,820	39.1	39.1	.000657	.0377	482
0.80	21,900	50.8	50.8	.000657	.0377	515

3.4 Predicted amplitudes of oscillation using the methods of para.2.

Shannon's Method.

Because the dynamic magnifier predicted by Shannon's equation (eqn.No.7) is dependent solely on the shape of the elastic curve (normal mode) and this does not change appreciably in the Ford Zephyr rig for any of the assemblies tested the method gives $M \doteq 10$ for all the arrangements tested.

Therefore the predicted amplitude for any intermediate shaft in the rig fitted with all six pistons is :-

$$= \pm 1.68 \text{ deg.}$$

which is roughly 80% the actual values experienced.

The method gives the same value of $M \doteq 10$ for the rig when only No.1 piston is fitted, therefore the predicted amplitudes are:-

$$\text{2nd order vibration} = \pm 1.853^\circ$$

$$\text{3rd order vibration} = \pm 0.377^\circ$$

respectively for all 5 intermediate shafts. These results are approximately correct in the case of the 2nd order and one fifth in the case of the 3rd order of the actual results.

While the rig in this form is not a normal engine this does serve to show the complete lack of general application of this method.

Ker Wilson's Method.

$$M = \frac{500}{\sqrt{\tau_0 + 16}} \qquad \text{Eqn. no. ... (8)}$$

Applied to the rig with 6 pistons and each of the five intermediate shafts in turn :-

Intermediate shaft	τ_0	M	$\pm\ \theta_1$ deg
0.60	1782	11.8	1.98
0.65	1930	11.3	1.90
0.70	2065	10.96	1.83
0.75	2190	10.65	1.79
0.80	2340	10.3	1.73

These predicted values are very close to the observed values.

Applied to the rig with No.1 piston only for 2nd order:-

Intermediate shaft	τ_0	M	$\pm\ \theta_1$ deg
0.60	1930	11.3	2.1
0.65	2090	10.85	2.01
0.70	2290	10.5	1.95
0.75	2380	10.2	1.89
0.80	2540	9.9	1.83

which again,are very close to observed values.

and for 3rd order:-

Intermediate shaft	τ_0	M	$\pm\ \theta_1$ deg
0.60	393	24.8	0.935
0.65	424	23.8	0.898
0.70	454	23.0	0.867
0.75	482	22.4	0.845
0.80	515	21.7	0.820

which are about half the observed values.

Draminsky's Method.

$$p = (.004\, i^2 + .06)n + 0.6 \quad \text{per cent .. Eqn.no. (13)}$$

where

$p = 1.212$ (3rd order) $\qquad n = 3$ (3rd order)

or $p = 1.008$ (2nd order) $\qquad n = 2$ (2nd order)

$\therefore$ M (3rd order) = 82.5

M (2nd order) = 99.3

giving for the rig with 6 pistons $\quad \theta_1 = \pm 13.85^\circ$

for the rig with No.1 piston only $\quad \theta_1 = \pm 3.11^\circ$ 3rd order

and $\quad \theta_1 = \pm 18.4^\circ$ 2nd order

Draminsky's formulae were intended for large marine engines and appear to be completely unsuitable for the Ford Zephyr. While the 3rd order results for the rig with No.1 piston only are double, the others are 6 and 8 times the observed values.

Measured values of dynamic magnifier.

Since the overall engine dynamic magnifier is so widely used it is of interest to determine it for the tests completed on the rig.

Rig with 6 pistons.

Removing the piston rings had only a slight effect, the values below are for all pistons with all piston rings, 3rd order vibration.

Intermediate Shaft	M
.60	*
.65	11.96
.70	11.3
.75	*
.80	11.68

Most normal complete engines fall between 20 and 40.

Rig with No.1 piston and connecting rod only.

3rd order.

Intermediate Shaft	M
.60	47.0
.65	44.3
.70	42.5
.75	47.5
.80	38.7

2nd order.

Intermediate Shaft	M
.60	*
.65	11.7
.70	11.05
.75	*
.80	9.8

*peak amplitudes above the guaranteed linearity of the transducer.

4. RESULTS OF THE TESTS OF THE RIG

The lubricating oil used in all tests was Mobiloil Special for which the viscosity - temperature chart is shown in Fig.19. The oil pressure in the main oil pipe before the bearings was 52 to 55 lbf./in^2. When no oil heater was fitted the temperature stabilised at approximately 88°F and unless otherwise stated the test readings were made when this temperature was reached.

The first series of tests which was carried out involved changing the temperature of the lubricating oil in a series of steps as it was thought that this would show an important order of change in the damping.

During these tests the 0.65" diameter intermediate shaft was fitted. There were 6 pistons but no piston rings. The results were :-

Lubricating oil temperature $^{\circ}$F	Maximum amplitude $\pm$ deg.
88	2.065
92	2.065
104	2.065
110	2.00
120	2.00
130	2.00

At this stage an N.E.P. ultra violet recorder was being used with 4 in wide recorder paper and there was only a limited selection of galvanometers. $\pm$ 2.065 deg. corresponded to a diagram height of 1.55 in. and 2.00 deg. corresponded to 1.50 in. It was concluded that within the capacity of the recording system there was no measurable change in amplitude. The very slight apparent drop in amplitude with increased oil temperature is the reverse of what was expected especially as the tests were carried out in the order shown.

In subsequent tests a Southern Instruments U.V. recorder type M.1500 was used with six inch wide paper and a better range of galvanometers.

The type of record taken at constant speed is shown in Fig.6.

Three traces were recorded simultaneously :-

1. The signal from the torsiograph.
2. A once per revolution marker at No.1 TDC.
3. Timing marks at 1/10th sec. interval.

All the earlier records taken at constant speed were checked by a 'run down' record taken as the speed of the engine gradually dropped through the critical after the clutch was withdrawn. One of these records is shown in Fig.7. This practice was discontinued later as the 'run down' record duplicated the constant speed reading. Instead a 'constant speed' type of reading was taken while the speed of the rig was allowed to rise gradually through the critical in case the exact peak reading had been missed slightly.

The next series of tests was carried out with all 6 pistons fitted:

(a) with no piston rings,

(b) with all 18 piston rings.

(a) 6 pistons, no piston rings.

TABLE 5 .

Inter. shaft in.dia.	Calculated natural frequency. rad./sec.	Measured natural frequency rad./sec.	3rd order amplitude at No.1 end	
			deg.	rad.
0.60	238	238	*	
0.65	279	279	2.065	.0360
0.70	323	311	1.945	.0339
0.75	370	361	*	
0.80	420	395	2.016	.0352

(b) 6 pistons, 3 piston rings in each piston

TABLE 6 .

Inter. shaft in.dia.	Calculated natural frequency. rad./sec.	Measured natural frequency rad./sec.	3rd order amplitude at No.1 end	
			deg.	rad.
0.60	238	238	*	
0.65	279	279	2.01	.0351
0.70	323	311	1.90	.0332
0.75	370	361	*	
0.80	420	395	1.96	.0342

* peak reading above the guaranteed linear response of the transducer.

The full results are shown on Fig.16.

For the third series all pistons and connecting rods except those in No.1 line were removed and a series of tests with 1 ring (top), 2 rings (top and 2nd) and 3 piston rings in No.1 piston was carried out. The results are shown in tabular form in Table 7 and fig.17, & 18.

With only one piston in the rig there is also a 2nd order torque harmonic which could easily be recorded but in a number of cases the amplitude was above the known limit of accuracy for the transducer (see response characteristics later). Accuracy can be guaranteed up to $\pm 2.04^{\circ}$ though it may extend higher.

TABLE 7.

EFFECT OF THE PISTON RINGS ON AMPLITUDE OF OSCILLATION

No.1 piston only

Shaft in.dia	Calculated natural frequency rad./sec.	Measured natural frequency rad./sec.	± deg. Measured amplitude 3rd order				± deg. Measured amplitude 2nd order			
			0 RINGS	1 RING	2 RINGS	3 RINGS	0 RINGS	1 RING	2 RINGS	3 RINGS
0.60	253	260	2.04	1.83	1.71	1.77	*	*	*	*
0.65	297	300	1.95	-	1.68	1.67	2.42*	2.20*	2.17*	2.17*
0.70	344	340	1.71	1.67	1.64	1.60	2.14*	2.06*	1.99	2.05
0.75	394	389	2.19*	2.10*	2.06*	1.79	*	*	*	*
0.80	448	444	1.62	1.66	1.47	1.46	2.03	1.92	1.90	1.82

These results are shown in graph form on graphs Nos. 17 and 18.

* above guaranteed limit of accuracy.

5. HYSTERETIC DAMPING

Internal friction or hysteretic damping has been studied by many but the work of Hatfield (3) on a wide variety of metals and that of Dorey (9) on crankshaft steels seems particularly relevant.

Within the fatigue limit of the material the damping work done per cycle $\Delta W = A\tau^{m}$ per unit vol. where τ is the uniform shear stress and A and m are constants depending on the material, its heat treatment and previous stress history.

For a hollow circular shaft the damping loss per cycle becomes

$$\Delta W = A\frac{2\pi\tau^{m}}{r_1^{m}}\left[\frac{r_1^{m+2} - r_2^{m+2}}{m+2}\right] \quad \text{work/cycle/unit length}$$

where (τ = max. shear stress
(r_1 = outer rad.
(r_2 = inner rad.

and for a solid circular shaft this reduces to

$$\Delta W = A\frac{2\pi\tau^{m}r_1^{2}}{m+2} \quad \text{work/cycle/unit length} \ldots \quad \ldots \quad \ldots (14)$$

The factor A and exponent m were obtained for the intermediate shafts experimentally. Two methods were used and gave the same result.

(a) A flywheel, intermediate shaft and crankshaft assembly with a torsional vibration transducer was hung on a bifilar suspension and oscillated in its torsional mode freely while the decaying amplitude was recorded on a U.V. recorder, see photograph No.7.

(b) With the same assembly the flywheel was clamped rigidly to a roof beam and the same method of recording used, photograph No.6.

The first method using the bifilar system was preferred after reasonable agreement was obtained between the two and all the results reported were obtained on the bifilar suspension. Damping was always slightly less on the bifilar suspension, this can be seen in Fig.12 where both methods are recorded. In order to obtain damping values at stress in the order of 10 tonf/in^2 which was being obtained in the Ford Zephyr rig it was necessary to 'waist' a spare shaft which had been made in the same way as the others, see photograph No.8.

Measured damping for the intermediate shafts and for the $\frac{3}{8}$" dia. waisted shaft are shown in Figs.11 and 12.

The log decrement $\delta = \frac{1}{n} \log_e \frac{x_1}{x_{(1+n)}}$

where x = amplitude

n = no. of cycles

and $2\delta\left(1 - \delta + \frac{2}{3}\delta^2 - \frac{1}{3}\delta^3 + \cdots\right) = \frac{\Delta W}{W}$ (see appendix 2).

where ΔW = work/cycle/unit length

W = maximum strain energy per unit length.

at low values of damping

$$2\delta \simeq \frac{\Delta W}{W}$$

$$W = \int_0^{r_1} \frac{\tau_r^2\, 2\pi r}{2G}\, dr$$ per unit length of circular shaft radius r_1

$$= \frac{\tau^2 \pi r_1^2}{4G}$$ per unit length

$$\therefore \Delta W = \frac{\delta \tau^2 \pi r_1^2}{4G}$$ work/cycle/unit length

... ... (15)

The measured damping loss per unit length of the five intermediate shafts is shown in Fig.11. It proves to be exactly linear on a log-log plot. The maximum stress reached in the decrement tests on the five shafts was in the order of 2 tonf/in^2 but a maximum stress of $\pm$ 14.75 tonf/in^2 was reached in a similar test on the waisted specimen and proved also to be linear over this range, see Fig.12. Previous to having the decrement tests carried out on them all of the five intermediate shafts had been run in the rig at the operating stress of the critical. The Fig.13 shows the results of the tests up to 2 tonf/in^2 extrapolated to cover the stress range met in the rig tests. The Fig.14 refers the results obtained on the $\frac{3}{8}$" diameter specimen to the larger diameter of the intermediate shafts. It was considered that the test on the $\frac{3}{8}$" diameter specimen which showed the lowest specific damping of all would be the most correct as any experimental errors would be expected to increase and not to reduce the observed damping loss. The $\frac{3}{8}$" specimen needed much less torque to excite it initially and would therefore be less likely to suffer interface friction losses. Air friction is considered to be a negligible effect in these tests.

A further comparison is given by Fig.15 which shows the calculated damping for the intermediate shafts based on the data in Hatfield's report (ref:3) for a 0.21%C normalized steel specimen 5/16 in. diameter.

The steel used in making the intermediate shafts was used to make a Hounsfield Tensometer test piece 0.252 in. diameter and a torsion test piece.

The results of the tension test were :-

Yield point	23 tonf/in.^{2}
Ultimate tensile strength	31 "
Elongation	28%
Reduction of area	50%

These results and the microscopic structure, photograph No.10, are typical of a 0.18 to 0.20%C steel normalized.

The experimentally obtained values of the constant A and exponent m in the expression $\Delta W = A\tau^m$ are listed below:-

TABLE 8

Measured factors for damping energy loss of the intermediate shafts

Shaft	A	m	Shear stress range tonf/in^2
0.60	$\frac{1}{443}$	2.53	0.5 to 2.0
0.65	$\frac{1}{265}$	2.40	"
0.70	$\frac{1}{130}$	2.45	"
0.75	$\frac{1}{1000}$	2.46	"
0.80	$\frac{1}{114}$	2.405	"
0.375	$\frac{1}{115}$	2.02	1.9 to 8.3
"	$\frac{1}{212}$	2.21	8.0 to 14.75
0.3125 (Hatfield)	$\frac{1}{348}$	2.41	2 to 10

The very low damping recorded for the 0.75 shaft is reflected in the high amplitudes obtained in the rig when running with that shaft. In most cases this has resulted in the amplitude of oscillation being above the limit of linear response of the transducer and all results with this shaft fitted have been eliminiated.

This serves as a useful indication of the vagaries of hysteretic damping because every reasonable care was taken in making these shafts but they were ordinary bar stock and were normalized commercially where no direct supervision was possible.

Log decrement test on $\frac{3}{8}$" diameter waisted intermediate shaft on bifilar suspension.

Natural frequency of oscillation 36.5 cps ω = 230 rad/sec.

M_1 = 0.417 lb.in.sec^2

M_2 = 0.548 lb.in.sec^2

K = 12,500 lbf in./rad.

The effective length of the waist = 1.86 in.

TABLE 9

Calculation of damping loss from the decrement tests. $\frac{3}{8}$" dia.shaft

θ_1	θ_{1+n}	$\log_e \frac{\theta_1}{\theta_{1+n}}$	n	τ_{max} tonf/in^2	δ	W lbf in/in	ΔW
.886	.740	.1790	9	14.75	.0199	2.51	.100
.740	.620	.1781	9	12.3	.0198	1.745	.0692
.620	.530	.1570	9	9.75	.0174	1.096	.0382
.530	.450	.1621	9	8.83	.018	.90	.0324
.500	.415	.185	9	8.32	.0205	.80	.0328
.400	.330	.191	9	6.66	.0205	.513	.021
.300	.250	.182	9	5.00	.0205	.289	.01185
.200	.167	.182	9	3.33	.0205	.128	.00525

The log decrement results are given below for the 0.60 shaft and the 0.80 shaft only.

0.60 shaft, natural frequency of oscillation = 51 cps

$M_1 = 0.417$ lb.in.sec.^{2}

$M_2 = 0.548$ lb.in.sec.^{2}

TABLE 10

Calculation of damping loss 0.60 shaft

θ_1	θ_{1+n}	$\log_e \frac{\theta_1}{\theta_{1+n}}$	n	τ_{max} tonf./in^2	δ	W lbf in/in	ΔW
.238	.220	.0779	$5\frac{1}{4}$	1.745	.0061	.0895	.001091
.200	.188	.0618	$5\frac{1}{4}$	1.465	.00485	.063	.000612
.150	.141	.0618	$5\frac{1}{4}$	1.100	.00485	.0356	.000346
.100	.096	.0400	$5\frac{1}{4}$	.7325	.00314	.0158	.0000991
.050	.0486	.0283	$5\frac{1}{4}$	.366	.00222	.00394	.0000175

0.80 shaft, natural frequency of oscillation = 84 cps.

TABLE 11

Calculation of damping loss 0.80 shaft

θ_1	θ_{1+n}	$\log_e \frac{\theta_1}{\theta_{1+n}}$	n	τ_{max} tonf/in^2	δ	W lbf in/in	ΔW
.1905	.115	.507	21	1.86	.02415	.1818	.00878
.15	.095	.457	21	1.465	.0268	.113	.00606
.10	.068	.385	21	.975	.0183	.0499	.00183
.05	.0372	.2965	21	.487	.0141	.01246	.0003515
.025	.0204	.203	21	.244	.00966	.00313	.0000605
.020	.0165	.1916	21	.195	.00912	.0020	.0000364

6. OIL DAMPING AT THE BEARINGS

Estimation of the energy loss in oil damping is complicated by the difficulties of predicting the movements of the journals of a crankshaft during twisting.

Shearing of the oil film between the journal and bearing in a flooded main or big end due to rotations is calculable but does not account for all the energy loss. Of probable greater importance is the energy dissipation due to transverse motion of the journals leading to a circumferential movement of oil round the annulus.

Shannon (4) considered the rate of energy dissipation due to transverse movements of a journal and after some approximations arrived at an expression for the energy loss which will be compared with an analysis based on the Reynold's equation.

6.1 Damping due to shearing of the oil film.

Consider the simple shear stress of the oil film due to cycle rotation of a circular journal in an oil filled bearing. The simple Petroff expression for frictional force at the periphery of a bearing assumed to be full of oil of one viscosity is

$$F = \eta \frac{dU}{dr} \times \text{bearing area}$$

$$= 2\pi\eta \frac{U r L}{c_r} \qquad \ldots \quad \ldots \quad \ldots (16)$$

If the offset or eccentricity is taken into account

$$F = \frac{c_r \varepsilon W}{2r} \sin\psi + \frac{2\pi\eta\, U r L}{(1-\varepsilon^2)^{\frac{1}{2}}\, c_r} \qquad \ldots \quad \ldots \quad \ldots (17)$$

(ref.7 p.296)

where ε is the eccentricity ratio.

ε	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	.95
$\frac{1}{(1-\varepsilon^2)^{\frac{1}{2}}}$	1	1.005	1.02	1.05	1.08	1.15	1.25	1.40	1.67	2.29	3.20

The oil used throughout the tests was Mobiloil Special and a graph of its viscosity against temperature is shown in Fig.19.

All the tests recorded were conducted at oil temperatures in the system of about 86 - 90°F. at which the viscosity is 1.305×10^{-6} reyns.

TABLE 12

The bearing dimensions

	Diameter of bearing in.	Width in.	Central groove in.	diameter of journal in.
Flywheel end main	2.378	1.35	0.2	2.376
Nos. 4, 5 main	2.378	1.10	0.2	2.376
Nos. 2, 3 main	2.378	1.10	0.2	2.376
Free end main	2.378	0.97	0.2	2.376
Big end bearing	2.128	1.10	-	2.126

Eccentricity ratio of the main journals.

The Sommerfeld equation (ref.7, p.285) is a simple means of obtaining an approximation to the eccentricity ratio.

$$\frac{W/L \quad c_r^2}{U \eta \quad r^2} = \frac{6\pi\varepsilon}{(1-\varepsilon^2)^{3/2}\,(1+\frac{\varepsilon^2}{2})} \quad \ldots \quad \ldots \quad \ldots (18)$$

Where L = axial length of bearing, say
3.72 in. for all four main bearings.

W = total load

The unbalance W.R. of the crankshaft is calculated to be

Nos.1, 2, 5 and 6 = 1.411 lbf.in.
Nos. 3 and 4 = 1.708 lbf.in.

The unbalanced W.R. of the connecting rod per line is

$$1.156 \times 1.565 = 1.810 \text{ lbf.in.}$$

but this is balanced when all pistons are fitted.

The rotating force is therefore $\dfrac{2(1.708 - 1.411)\omega_f^2}{386}$

$$= .00154\,\omega_f^2 \quad \text{lbf.}$$

Consider the rig with 6 pistons, 3rd order critical

	ω_f^2	rotating force lbf	weight of crankshaft lbf
0.60	6400	9.85	74.2
0.65	8640	13.30	74.2
0.70	10800	16.60	74.2
0.75	14540	22.40	74.2
0.80	17350	26.70	74.2

These small forces lead to a negligible eccentricity at which $\psi \simeq 90^\circ$.

therefore $\quad \dfrac{c_r\, \varepsilon\, W}{2r} \sin\psi \simeq 0$

and $\quad \dfrac{1}{(1-\varepsilon^2)^{\frac{1}{2}}} \simeq 1$

The Petroff equation will therefore be used in its simplest form here but eccentricity ratios in the region 0.6 to 0.8 are common in dynamically loaded bearings and would then need to be taken into account.

$$F = \frac{2\pi \eta U r L}{c_r}$$

where $\quad U = \omega_n \theta r \cos\omega t$

$r = 1.188$ in. (main bearing)

$r = 1.063$ in. (big end bearing)

$L = 3.72$ in. (four main bearings)

$L = 1.10$ in. (one big end bearing)

$C_r = .001$ in.

F is therefore a function of $\cos\omega t$.

The work done per cycle = $F \pi \theta r$

$$= \frac{2\pi^2 \eta \omega \theta^2 r^3}{c_r} \quad \ldots \quad \ldots (17)$$

$\therefore$ at all four main bearings work per cycle = $1.6\omega\theta^2$ lbf.in/cycle

and at each big end bearing work per cycle = $0.34\omega\theta^2$ lbf.in/cycle

The reciprocating force per line is

$$W_p = \frac{1.61}{386} \omega_f^2 (1.565)^2 \times 1.294$$

$$= .0134 \omega_f^2 \text{ lbf.}$$

where only No.1 piston is fitted the revolving force per line is

$$W_R = \frac{1.81}{386} \omega_f^2 (1.565)^2$$

$$= .0115 \omega_f^2 \text{ lbf.}$$

6.2 Damping due to transverse movement of journals

Shannon (ref.4) derived an expression for the rate of dissipation of energy per unit width of a journal in an oil filled bearing

$$F = \frac{\eta f^2 \omega_n^2 \pi^3 d^3}{10 c_r} \qquad \ldots \qquad \text{Eqn. no. (6)}$$

where f = eccentricity ratio e/cr of transverse movement

d = diameter of bearing

at the main bearings:

$\eta = 13.05 \times 10^{-6}$ reyns.

$d = 2.376$ in.

L = length of 4 main bearings = 3.72 in. effectively

$C_r = .001$ in.

$\therefore$ Work done per cycle for all four main bearings

$$= F.\frac{2\pi}{\omega}.L.$$

$$= 12.7\, f^2 \omega_n \text{ lbf in/cycle.}$$

and at the big end bearings:

$\eta = 13.05 \times 10^{-6}$ reyns

$d = 2.126$ in.

$L = 1.10$ in. (each big end bearing)

$C_r = .001$ in.

Work done per cycle per big end bearing = $2.69\, f^2 \omega_n$ lbf in/cycle.

Damping at the Main Bearings

In reference 7, p.357, are derived orthogonal damping force terms for a steadily loaded bearing due to transverse motion of the journal.

They only give a correct order of magnitude but in reference 8 some further computations indicate the degree of approximation.

The damping terms are given in the form

$$F_x^1 = \frac{-W}{g}\left(b_{xx}\dot{x} + b_{xz}\dot{z}\right)$$

$$F_y^1 = \frac{-W}{g}\left(b_{zz}\dot{z} + b_{zx}\dot{x}\right) \qquad \ldots \quad \ldots \quad \ldots (18)$$

where W is assumed to be a gravitational load and

$$b_{xx} = \frac{g}{c_r \omega}\left(\frac{2\sin\psi}{\varepsilon}\right) \qquad b_{zz} = \frac{g}{c_r \omega}\left(\frac{2\sin\psi}{\varepsilon}\right)$$

$$b_{xz} = \frac{g}{c_r \omega}\left(-\frac{2\cos\psi}{\varepsilon}\right) \qquad b_{zx} = \frac{g}{c_r \omega}\left(\frac{2\cos\psi}{\varepsilon}\right) \qquad \ldots \quad \ldots (19)$$

Damping of this type may occur at the main bearings or at the big ends.

For small eccentricities $\sin\psi \simeq 1$

$$\frac{W/L \; c_r^2}{U\eta r^2} \simeq 6\pi\varepsilon$$

$$\therefore \; W = \frac{6\pi\varepsilon U \eta r^2 L}{c_r^2}$$

$$\therefore \; F_y' = \frac{W}{g} b_{zz} = \frac{12\pi U \eta r^2 L}{c_r^3 \omega} \qquad \ldots \quad \ldots \quad \ldots (20)$$

The cross terms will $\rightarrow 0$ because $\psi \simeq 90^\circ$. $\dot{z}$ for the pin = $\theta \omega R \cos wt$ but what matters is the relative velocity between pin and journal. This is unknown but the relative amplitude of transverse movement cannot be greater than $c_r \cos wt$ $\therefore$ assume the transverse movement is fc_r maximum. The velocity is then $f\, c_r\, \omega_n \sin \omega_n t$

$$\text{Work done per cycle} = f^2 c_r^2 \omega_n F\, y' \pi \qquad \text{at resonance}$$

$$= \frac{12\pi^2 U \eta r^2 L f^2 \omega_n}{c_r \omega} \quad \ldots (21)$$

at the main bearings $\eta = 13.05 \times 10^{-6}$ reyns.

$U = r.W.$

ω = angular velocity of crankshaft

$r = 1.188$ in.

$L = 3.72$ in. total

ω_n = frequency of oscillation

$C_r = .001$ in.

the work done per cycle at all four main bearings $= \dfrac{12\pi^2 \eta r^3 L f^2 \omega_n}{c_r}$

$= 9.6\, \omega_n f^2$ lbf in.

which compares with $12.7\, \omega_n f^2$ by Shannon's expression.

For a single big end where:-

$r = 1.063$ in.

$L = 1.10$ in.

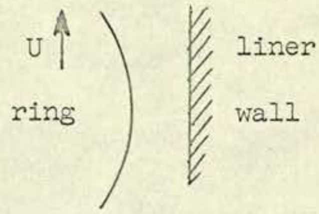
$c_r = .001$

the work done per cycle $= 2.05\, \omega_n f^2$ lbf in.

which compares with $2.69\, \omega_n f^2$ by Shannon's expression.

7. PISTON RING DAMPING

Consider the ring as presenting a curved face to the cylinder liner wall.



The equation for flow in one dimension

$$q = \frac{Uh}{2} - \frac{h^3}{12\eta}\frac{dp}{dx}$$

= carried flow - pressure flow

(reference 7, p.421)

$$\text{or } \frac{dp}{dx} = \frac{6U\eta}{h^2} - \frac{12}{h^3}\eta q$$

where q = flow of lubricant in x direction per unit width

U = surface speed in x direction

η = viscosity

h = film thickness

$\frac{dp}{dx}$ = pressure gradient

the frictional drag per unit area of moving surface is

$$\tau = \frac{\eta U}{h} + \frac{h}{2}\frac{dp}{dx}$$
$$= \frac{4\eta U}{h} - \frac{6}{h^3}\eta q$$

the drag per unit width is then

$$F = 4\eta U\int_{x_1}^{x_2}\frac{dx}{h} - 6\eta q\int_{x_1}^{x_2}\frac{dx}{h^2} \qquad \ldots \quad \ldots (22)$$

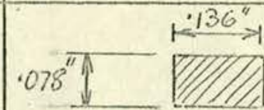
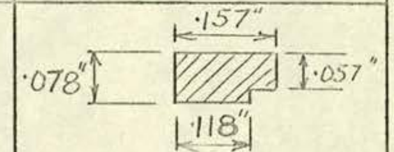
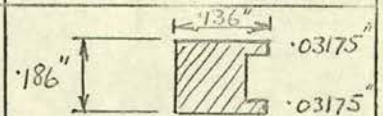
Determination of h the oil film thickness analytically will need a solution of the Reynold equation and experimentally it would need special instrumentation. It will certainly vary with the piston speed, and therefore with the position of the piston.

The difficulties here are considerable because conditions in a real engine are likely to be different on the upward and downward strokes and also between a downward firing and non-firing stroke.

However, the observed fact in the rig that the combined damping of all 18 piston rings in an engine where the amplitude of oscillation at the pistons is high in proportion to that normally encountered but where the rings provide only 3% of the damping loss indicates that this is not a source of damping which it will be profitable to examine very deeply.

TABLE 13

DETAILS OF THE PISTON RINGS

	face width in.	radial depth in.	I $in.^4$	Diametral force to close gap: W CALCULATED lbf	Diametral force to close gap: W_t TEST lbf	Nominal face pressure $lbf/in.^2$ $\frac{0.76\ W_t}{Db}$	SECTION OF RING
TOP RING	0.078	0.136	16.4 $\times 10^{-6}$	10	9.8	29.4	
SECOND RING	0.078	0.157	21.1 $\times 10^{-6}$	12.5	9.5	39	
SCRAPER	2 x .03175	0.136	34.5 $\times 10^{-6}$	12.7	9.9	37	

$E = 16.5 \times 10^6\ lbf/in.^2$

initial gap = 0.5 in.

closed gap = 0.012 in.

diameter of cylinder = 3.250 in.

8. DETERMINATION OF THE DAMPING LOSSES

8.1 With six pistons fitted and no piston rings, assuming the change of phase due to damping to be negligible, see Appendix 2, the total work done by damping per cycle $\sum W$, is as follows:-

Shaft	ω_n	3rd order amplitude $\pm\theta$ rad	deg	$\sum Q$ lbf in.	$\sum W$ lbf in. $=\sum Q\theta\pi$
0.65	279	.036	2.065	119.8	13.6
0.70	311	.034	1.945	139.2	14.9
0.80	395	.0352	2.016	248.5	26.4

The oil damping due to shearing can be calculated and the decrement tests provide data on the hysteretic damping.

ΔW_h = hysteretic damping from fig.13

ΔW_{mb} = damping due to oil shear at four main bearings .(Eqn.21)

ΔW_{be} = damping due to oil shear at six big end bearings. (Eqn. 21)

Shaft	ΔW_h	Lbf in/cycle ΔW_{mb}	ΔW_{be}
0.65	.945	.580	.740
0.70	1.115	.575	.735
0.80	2.09	.785	.995

The following energy losses are still unaccounted for:-

	lbf.in/cycle
0.65	11.33
0.70	12.57
0.80	22.53

This may be associated with movement of the big ends or the main bearings ∴ either $9.6\,\omega_n f^2$ lbf.in/cycle or $12.7\,\omega_n f^2$ lbf.in/cycle may be used to discover an order of amplitude of a transverse movement of the journals which would absorb this energy ΔW_{tr}

If $\Delta W_{tr} = 12.7\,\omega_n f^2$ then the movement e required is:-

	f^2	f	e in.
0.65	.0032	.0566	$.0566 \times 10^{-3}$
0.70	.00318	.0565	$.0565 \times 10^{-3}$
0.80	.00449	.0670	$.067 \times 10^{-3}$

8.2 With six pistons fitted and all piston rings the results are:-

Shaft	ω_n	3rd order amplitude $\pm\,\theta_1$ rad	deg.	ΣQ Lbf.in.	$\Sigma W = \Sigma Q \theta \pi$
0.65	279	.035	2.01	119.8	13.2
0.70	311	.0331	1.90	139.2	14.5
0.80	395	.0342	1.96	248.5	25.34

Using the same symbols as before and units of lbf in/cycle for the ΔW.

	ΔW_h	ΔW_{mb}	ΔW_{be}
0.65	.81	.548	.700
0.70	1.075	.545	.695
0.80	1.925	.740	.944

The loss which in the previous test was ascribed to pumping loss at the main bearings and big ends will be reduced in the probable ratio of the θ_1^2, therefore the loss from this source in this test becomes:-

	ΔW_{tr}
0.65	10.75
0.70	11.98
0.80	21.30

leaving as the energy lost at the piston rings.

	W_{pr}
0.65	.392
0.70	.205
0.80	.431

The approximate % energy loss can now be set out for the 3 assemblies when the rig has 6 pistons and all 18 piston rings.

TABLE 14

Inter. shaft	Hysteresis	Shear at MB	Shear at BE	Oil pumping at MB & BE	Piston rings
0.65	6.13	4.15	5.3	81.45	2.97
0.70	7.42	3.76	4.8	82.6	1.42
0.80	7.60	2.93	3.73	84.04	1.70

% Energy Loss

8.3 When fitted with only No.1 piston and no piston rings:-

Inter. Shaft	ω_h	3rd order amplitude $\pm\theta_1$ rad.	deg.	ΣQ lbf.in.	ΣW
0.60	260	.0356	2.04	17.44	1.95
0.65	300	.034	1.95	23.2	2.47
0.70	340	.0298	1.706	29.8	2.79
0.75	389		*		
0.80	444	.02825	1.62	50.8	4.51

Using the symbols as before but remembering that there is now only 1 big end bearing though still 4 main bearings:-

	ΔW_h	$\Delta W_{mb}(4)$	$\Delta W_{be}(1)$
0.60	.540	.527	.112
0.65	.675	.555	.118
0.70	.810	.483	.103
0.75			
0.80	1.215	.567	.1202

Subtraction of ΔW_h, ΔW_{mb} and ΔW_{be} from ΣW leaves:-

	W_{tr}
0.60	.771
0.65	1.122
0.70	1.394
0.75	
0.80	2.608

When these results are compared with the corresponding tests with all 6 pistons fitted and no piston rings it can be seen that there has been a considerable drop in the damping. This is apparent without any analysis of the figures because the input torque has dropped to 1/5th of that with all pistons yet the amplitude of vibration in all cases has scarcely dropped at all. The oil pressure to the main bearings did not fall due to the absence of 5 connecting rods therefore the remaining big end and 4 main bearings must have continued to be flooded with oil and therefore subject to shearing losses.

The apparent inference is that the missing 5 connecting rods and pistons must somehow be responsible for the damping which has disappeared simultaneously with them.

The damping due to one set of 3 rings on No.1 piston can now be determined:-

8.4 When fitted with No.1 piston only and all 3 piston rings

Shaft	ω_n	3rd order amplitude $\pm\theta_1$ rad	deg.	ΣQ lbf.in.	$\Sigma W = \Sigma Q \theta \pi$ lbf.in/cycle
0.60	260	.0309	1.77	17.44	1.695
0.65	300	.0291	1.67	23.2	2.12
0.70	340	.0279	1.60	29.8	2.61
0.75	-	-	-	-	-
0.80	444	.0254	1.46	50.8	4.05

	ΔW_h	$\Delta W_{mb}(4)$	$\Delta W_{be}(1)$
0.60	.472	.398	.0845
0.65	.554	.407	.0865
0.70	.709	.424	.0900
0.75	-	-	-
0.80	1.012	.458	.0975

lbf.in/cycle

Leaving, due to pumping action at bearings and the effect of the piston rings:-

	Δ W
0.60	.7405
0.65	1.0725
0.70	1.387
0.75	-
0.80	2.4825

The residual energy loss which is ascribed to oil movement at the bearings will now be reduced in the ratio of the θ_i^2, from the test with no piston rings in No.1 piston,

3rd order vibration:-

	ΔW_{tr}
0.60	0.59
0.65	0.821
0.70	1.22
0.75	-
0.80	2.11

If this is subtracted from the residual ΔW then what is left can be associated with piston ring damping.

	ΔW_{pr}
0.60	.1505
0.65	.2515
0.70	.167
0.75	-
0.80	.3725

The percentage breakdown of damping energy with only No.1 piston and 3 piston rings becomes:-

Inter. shaft	Hysteresis	Shear at MB	Shear at BE	Oil Pumping	Piston rings
0.60	27.8	23.5	5.0	34.8	8.9
0.65	26.2	19.2	4.0	38.8	11.8
0.70	27.2	16.2	3.5	46.7	6.4
0.75	-	-	-	-	-
0.80	25.1	11.3	2.4	52.0	9.2

TABLE 15

The removal of 5 of the connecting rods has had a considerable effect which can best be studied by comparing the energy losses ascribed to transverse movement of the journals (third order oscillations only) lbf.in/cycle.

TABLE 16

Inter shaft	Test 1		Test 2		Test 3		Test 4	
	ΔW_{tr}	$\pm \theta_1^{\circ}$	ΔW_{tr}	$\pm \theta_1^{\circ}$	ΔW_{tr}	$\pm \theta_1^{\circ}$	ΔW_{tr}	$\pm \theta_1^{\circ}$
0.60	-	-	-	-	.771	2.04	.59	1.77
0.65	11.3	2.065	10.75	2.01	1.122	1.95	.821	1.67
0.70	12.57	1.945	11.98	1.90	1.394	1.707	1.22	1.60
0.75	-	-	-	-	-	-	-	-
0.80	22.53	2.016	21.30	1.96	2.608	1.62	2.11	1.46

where test 1 was with all 6 connecting rods and pistons but no piston rings.

test 2 was with all 6 connecting rods, pistons and all 18 piston rings.

test 3 was with No.1 piston only, no piston rings.

test 4 was with No.1 piston only, and 3 piston rings.

As the amplitudes are very little different, especially for the 0.65 shaft between tests 2 and 3, this seems to indicate that most of these particular losses occur at the big ends of the connecting rods, though it is not certain how much that the pistons too may be involved.

8.5 2nd order oscillation.

It was not intended originally to investigate 2nd order effects as it was expected that amplitudes would be too high and it was known that the connecting rod couple introduces an additional 2nd order effect. However the amplitudes proved to be measurable in a few cases :-

Inter. shaft	ω_n	Test 5 $\pm\ \theta_1$ deg	Test 6 $\pm\ \theta_1$ deg
0.70	340	2.14	2.05
0.80	444	2.03	1.82

where test 5 was with No.1 piston only and no piston rings

test 6 was with No.1 piston only and 3 piston rings.

for test 5 the breakdown of damping losses leads to :-

	Hysteresis	Shear at MB(4)	Shear at BE(1)	Remainder ΔW lbf in/cycle
0.70	1.35	.762	.162	14.426
0.80	2.025	.892	.189	22.094

TABLE 17

8.6 Discussion of the results

Two effects of considerable magnitude can be observed from the results of the tests.

(1) Comparison between the 3rd order oscillation with 6 pistons fitted and the 3rd order oscillation with only 1 piston fitted shows a large drop in damping for which the connecting rods appear to be responsible.

(2) Comparison between the 3rd order and 2nd order response of the same engine build indicates an increase in damping during 2nd order vibration which is difficult to explain.

The damping due to transverse movement at the bearings during 2nd order oscillation with 1 piston only is almost exactly the same as that during 3rd order oscillation with all 6 pistons which would seem to contradict the findings of (1) above.

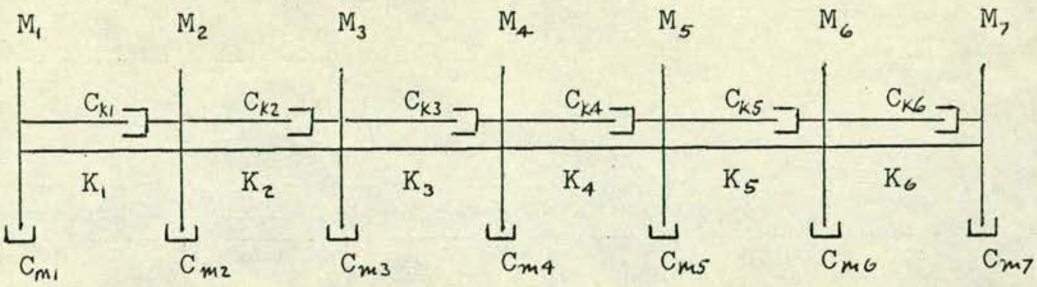
It is known that variations in the effective moment of inertia of the connecting rod have a 2nd order effect which has not been taken into account. This is examined in detail in Appendix 3 where it can be seen that the effect is appreciable but not sufficient to account for the differences observed. See also references 11 and 12.

The 3rd order results are not affected by the connecting rod couple effect and these results will be used to determine the damping coefficients.

9. DETERMINATION OF THE DAMPING COEFFICIENTS

9.1 Mathematical model of the system

The model of the system is:-



The equations of motion in general form are:-

$$\sum_n m_{ij}\ddot{\theta}_j + \sum_n c_{ij}\dot{\theta}_j + \sum_n k_{ij}\theta_j = F_j e^{j\omega_f t}$$

where F_j may be complex, but in the case of the 3rd order oscillation in the Ford Zephyr the F terms will be in phase, and when the 2nd or 3rd order is considered due to 1 piston only there is only one F term. (The usual assumption is made that the damping can best be expressed as viscous (an alternative for the structural damping is $m\ddot{\theta} + (1+ig)[k]\theta = Fe^{j\omega t}$))

The solution is assumed to be of the form

$$\theta_j = \boldsymbol{\theta}_j e^{j\omega_f t}$$

substituting for $\ddot{\theta}_j$ & $\dot{\theta}_j$ we get a set of linear equations for $\boldsymbol{\theta}_j$

$$\sum_n \boldsymbol{\theta}_j\left(-\omega_f^2 m_{ij} + j\omega_f c_{ij} + k_{ij}\right) = F_j$$

the $\boldsymbol{\theta}$ can be obtained as

$$\boldsymbol{\theta}_j = \frac{\Delta_j(\omega_f)}{\Delta(\omega_f)}$$

where $\Delta(\omega_f)$ is the determinant of the coefficients of $\boldsymbol{\theta}_j$ and $\Delta_j(\omega_f)$ is the same as $\Delta(\omega_f)$ except that the j^{th} column is replaced by F_1, F_2, -------F_n .

9.2 Formation of the mass, damping and stiffness matrices.

At resonance we can say with sufficient accuracy that $\omega_f = \omega_n$, the undamped natural frequency. The equations of motion may be formed by Lagrange's equation or by direct application of Newton's laws.

The mass matrix

$$[M] = \begin{bmatrix} m_1 & & & 0 \\ & m_2 & & \\ & & \ddots & \\ 0 & & & m_n \end{bmatrix}$$

The damping matrix $[C] =$

$$\begin{bmatrix} (C_{m1} + C_{K1}) & -C_{K1} & 0 & \cdots & 0 & 0 \\ -C_{K1} & (C_{m2} + C_{K1} + C_{K2}) & -C_{K2} & \cdots & 0 & 0 \\ 0 & -C_{K2} & (C_{m3} + C_{K2} + C_{K3}) & \cdots & 0 & 0 \\ & & \ddots & & & \\ 0 & 0 & \cdots & -C_{K(n-2)} & (C_{m(n-1)} + C_{K(n-2)} + C_{K(n-1)}) & -C_{K(n-1)} \\ 0 & 0 & \cdots & 0 & -C_{K(n-1)} & (C_{mn} + C_{K(n-1)}) \end{bmatrix}$$

and the stiffness matrix $\left[K \right]$ is

$$\begin{bmatrix} k_1 & -k_1 & 0 & 0 & - - - - - & 0 & 0 \\ -k_1 & k_1+k_2 & -k_2 & 0 & - - - - - & 0 & 0 \\ 0 & -k_2 & k_2+k_3 & -k_3 & & 0 & 0 \\ \vdots & & & \ddots & & & \vdots \\ 0 & 0 & 0 & - - - - & -k_{n-2} & k_{n-2}+k_{n-1} & -k_{n-1} \\ 0 & 0 & 0 & - - - - & 0 & -k_{n-1} & k_{n-1} \end{bmatrix}$$

In the rig it is assumed that

$C_{k1}, C_{k2}, \dots \dots .. C_{k5} = 0$ because the shear stresses are very low in the corresponding shaft positions. As No.7 is a flywheel only and virtually at a node the bearing supporting it is assumed to provide no damping and so $C_{m6} = 0$.

Since the difference of phase between the θ_n is very small at the levels of damping in engines, only θ_1 need by evaluated and its effective magnitude will be given by the imaginary term since it will to all intents and purposes occur $\frac{\pi}{2}$ out of phase with the torque at resonance.

In hysteretic damping the coefficient c_k which is used in the equations of motion is the equivalent viscous damping coefficient but hysteretic damping is not velocity dependent and it is necessary to denote c by $\frac{h}{\omega}$ where h is a constant for a particular shaft at a given amplitude of oscillation.

The energy dissipated per cycle for viscous damping is

$$\Delta W = \pi c \omega \theta^2$$

replacing c by $\frac{h}{\omega}$ gives

$$\Delta W_h = \pi h \theta^2$$ which is independent of frequency

referring to the tests with 6 pistons and no piston rings :-

	ΔW_h lbf.in.	h	c_k lbf.in/rad/sec.
0.65	.945	179	.642
0.70	1.115	239	.769
0.80	2.09	425	1.075

referring to the tests with 6 pistons and all piston rings :-

	ΔW_h	h	c_k
0.65	.81	162	.58
0.70	1.075	242	.778
0.80	1.925	415	1.050

referring to the tests with No.1 piston only and no piston rings:-

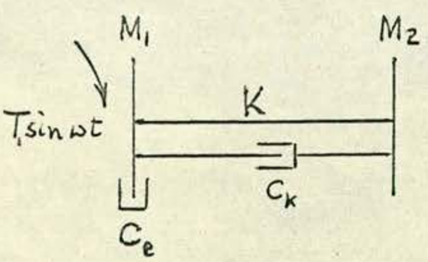
	ΔW_h	h	c_k
0.60	0.540	108	.415
0.65	.675	149.5	.498
0.70	.81	235	.692
0.75	-	-	-
0.80	1.215	400	.902

and referring to the tests with No.1 piston and all 3 rings :-

	ΔW_h	h	c_k
0.60	0.472	125.5	.483
0.65	.554	167	.556
0.70	.709	232	.682
0.75	-	-	-
0.80	1.012	410	.925

As a means of determining the overall engine damping coefficient and the order of phase difference when an experimentally observed value of θ_1 is available and shaft damping is known the method described above, para 9, can be used as follows:-

The model of the system is further simplified to



where M_1 = all engine masses

M_2 = flywheel mass (M_7)

$K = K_6$

C_e = all engine damping

$C_k = C_{k6}$ = intermediate shaft hysteretic damping

$T_1 \sin wt$ = engine harmonic torque = $T_1 e^{jwt}$

$$\Delta(\omega_f) = M_1 M_2 \omega_f^4 - j\omega_f\left\{c_e M_2 \omega_f^2 + C_K M_2 \omega_f^2 + c_K M_1 \omega_f^2\right\} + j\omega_f\left\{c_e K\right\} - \omega_f^2\left\{K M_2 + C_e C_K + M_1 K\right\}$$

but at resonance $\omega_f^2 \simeq \frac{K(M_1+M_2)}{M_1 M_2} = \omega^2$

$$\therefore \Delta(\omega) = -j\left(\frac{K(M_1+M_2)}{M_1 M_2}\right)^{3/2}\left\{c_e M_2 + c_K(M_1+M_2)\right\}$$

$$+ j\left(\frac{K(M_1+M_2)}{M_1 M_2}\right)^{1/2}\left\{c_e K\right\} - \frac{K(M_1+M_2)}{M_1 M_2} c_e c_K$$

$$= -j(\omega^3) c_e M_2 - j(\omega^3) c_K(M_1+M_2) + j\omega c_e K$$

$$- \omega^2 c_e c_K$$

An overall engine damping coefficient C_e may then be obtained because $\theta_1 = \frac{\Delta_1(\omega)}{\Delta(\omega)}$ is known from the experimental results.

Referring to the test with 6 pistons, no piston rings this results in the following values for C_e .

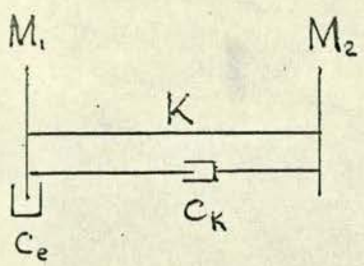
Intermediate shaft	Energy dissipated in engine Lbf in/cycle	C_e
0.65	12.65	11.1
0.70	13.78	12.28
0.80	24.31	15.8

Test with 6 pistons, all piston rings

0.65	12.79	11.85
0.70	13.82	12.88
0.80	24.47	16.85

Examination of the order of phase change brought about by the level of damping existing in the rig :-

The example used is the 0.65 shaft with all pistons and no piston rings. The idealised system is the extreme form :-



M_1 = all engine masses

= 0.4576 lb.in.sec^2

M_2 = flywheel

= 3.06 lb.in.sec^2

K = 31200 lbf in/rad

C_k = 0.642 lbf in/rad/sec.

C_e = 11.1 lbf in/rad/sec.

ω_f = 279 rad/sec.

Q = 119.8 lbf.in.

$$\Delta = 339{,}000 - j\,489.5 \times 10^{6}$$

$$\Delta_1 = Q\,(k - M_2\omega^2 + j\,\omega C_k) = 119.8\,(-207{,}800 + j\,179)$$

$$|\theta_1| = \frac{\Delta_1}{\Delta} = -17.7 \times 10^{-6} - j\,0.0361$$

The actual amplitude is ± 0.036 rad. the difference being slide rule error. The phase angle is seen to be less than 2 minutes greater than 90 deg. and it is known that the rig has damping slightly higher than is common in engine installations where the dynamic magnifier is usually in the range 20 - 40 .

This indicates the possibility of putting the damping matrix into a form that will allow the equations to be put into principal co-ordinates by a simple transformation. This becomes important when only a small, comparatively slow computer is available, such as the Elliot 803B .

In the generalized co-ordinates q , which in this problem will always be the amplitudes of oscillation at m_1, m_2 , etc.

$$\left\lceil m \right\rfloor \{\ddot{q}\} + [c]\{\dot{q}\} + [k]\{q\} = \{F(q,t)\}$$

$$\text{Let} \quad \{q\} = [\phi]\{p\}$$

where $[\phi]$ is the transformation matrix to principal co-ordinates for the undamped system.

i.e. $[\phi]$ is a matrix whose columns are the eigenvectors (normal modes) of the undamped equations

$$\text{then} \quad \{\dot{q}\} = [\phi]\{\dot{p}\}$$

$$\text{and} \quad \{\ddot{q}\} = [\phi]\{\ddot{p}\}$$

the equations now become:-

$$[\phi]^T \left\lceil m \right\rfloor [\phi]\{\ddot{p}\} + [\phi]^T[c][\phi]\{\dot{p}\} + [\phi]^T[k][\phi]\{p\} = [\phi]^T\{F(q,t)\}$$

or

$$\left\lceil M_r \right\rfloor \{\ddot{p}\} + [\phi]^T[c][\phi]\{\dot{p}\} + \left\lceil K_r \right\rfloor \{p\} = [\phi]^T\{F(q)\}f(t)$$

$$\text{but} \quad [\phi]^T[k][\phi] = \left\lceil \omega_r^2 \right\rfloor \left\lceil M_r \right\rfloor = \left\lceil K_r \right\rfloor$$

and if we compare those with $[\phi]^T[c][\phi]$ it will be seen that the latter will result in a diagonal matrix if (c) is proportional to either $\left[m \right]$ or $\left[k \right]$ or to a combination of both, i.e.

$$\text{if} \quad \left[c \right] = 2\beta \left[m \right]$$

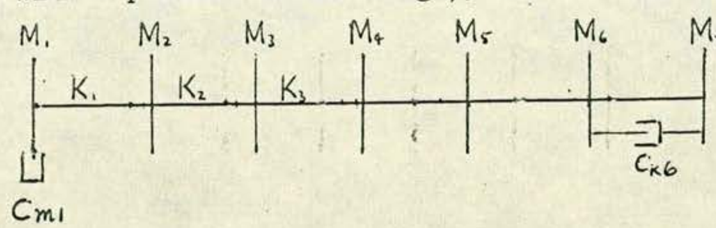
$$\text{or} \quad \left[c \right] = \alpha \left[k \right]$$

$$\text{or} \quad \left[c \right] = 2\beta \left[m \right] + \alpha \left[k \right]$$

(This was shown by Lord Rayleigh in "Theory of Sound" though not in matrix form).

The requirement now is to find out if the $\left[c \right]$ matrix can be put into one of these forms to absorb the same total energy as before. The rig used is typical enough in its distribution of damping to enable an examination to be made of this method.

Returning to the test result used before (of the 0.65 shaft with 6 pistons and no rings), a correct model is :-



	1	2	3	4	5	6	7
M	.08	.08	.0688	.0688	.08	.08	3.06 lb.in.sec^2
K	8.95	5.63	9.29	5.63	8.95	.031200 X 10^6 lbf in/rad.	
C_m	1.85	1.85	1.85	1.85	1.85	1.85	0
C_k	0	0	0	0	0	0.642	-

The C_m have been distributed equally at the engine masses to the required total of the engine damping.

If C_{k6} is transferred to C_{m7} to absorb the same energy then

$$C_{m7} = \frac{\Delta W_h}{\pi \omega \theta^2} = \frac{0 \cdot 945}{\pi . 279 . (\cdot 00535)^2}$$

$$= 37.6 \text{ lbf.in/rad/sec.}$$

This can be seem immediately to be of the correct order for one of the required relationships that $C_m = 2\beta \left[m \right]$

A re-distribution of damping coefficients to provide the required relationship but giving the same overall damping energy loss results in :-

C_m 1, 2, 5, 6 = 1.745 lbf.in/rad/sec.

C_m 3, 4 = 1.5 " " "

C_m 7 = 66.7 " " "

The problem can now be put into principal co-ordinates by the undamped normal mode transformation.

This method obviously has limited application and was explored only because of the excessively long time taken by the Elliot 803B computer to solve the correct equations, when the Runge-Kutta-Merson process was used.

10. METHODS OF SOLUTION OF THE EQUATIONS

It will always be useful to solve the undamped equations first to yield the natural frequencies and modal shapes. The undamped natural frequencies will not vary measurably from the damped frequencies.

As the unconstrained crankshaft problem will always give a stiffness matrix which is singular the flexibility matrix does not exist and it is customary to use the stiffness matrix $[K]$ and the diagonal mass matrix $[M]$ to form the characteristic matrix and to solve for the eigenvalues (natural frequencies) and the eigenvectors (modal shapes) by iteration.

$$K\{x\} = \omega^2 M\{x\}$$

$$M^{-1}K\{x\} = \omega^2\{x\}$$

$$\text{or} \quad A\{x\} = \lambda\{x\}$$

where $A = M^{-1}K$ is the characteristic matrix

$\lambda = \omega^2$ are the eigenvalues

$\{x\}$ = the eigenvectors, are the modal shapes.

The method used in the computer was an iterative process similar to the one described in reference 10, p.64, but the mode elimination was carried out by rows instead of as described by columns.

11. CONCLUSIONS

As much as 80% of the damping in torsional oscillation of an engine crankshaft may be caused by transverse movement of the journals, but further work is needed to measure the amount of this transverse oscillation to determine what parameters affect it and to find out why it differs for different orders of vibration.

Damping at the piston rings in the engine tested accounted for only 3% of the total energy lost in damping and will not in any normal engine be responsible for an appreciable quantity of damping.

Hysteretic damping (10% in the rig) will vary considerably depending on the stress level and shape of the more highly stressed components, but methods are available even for such irregularly shaped components as crankshafts (ref.13). Tests on a specimen of the material or published data can supply the necessary specific damping. Fatigue stress levels are kept so low (about 2 tonf/in^2.) in engine installations that this is not likely to be a major source of damping and on most installations it can probably be neglected.

Matrix methods of computation are attractive because use can be made of standard programs. The matrix iteration method of solving the characteristic equations for the eigenvalues and eigenvectors converges very rapidly to a solution and is much easier to program than the Holzer table which needs a complicated root finding process to make sure it converges.

The use of the damped equations of motion will not at this stage be an alternative method acceptable to industry for routine calculation of engines in production because the overall dynamic magnifier is readily obtainable for engines in service and is easily applied. Nevertheless, the method using a correct damping matrix is of considerable potential interest to designers and will become more useful if progress can be made with analytical assessment of the damping occurring at the bearings.

12. FUTURE WORK

The most useful research that could now be carried out in this field would be a fundamental study of damping at oil-filled bearings due to transverse oscillation of the journal.

In this connection the following experimental work could be carried out on the existing engine rig :-

(a) further comparative tests could be made with differing numbers and combinations of the connecting rods.

(b) different clearance bearings could be fitted (these are available)

(c) a free end flywheel could be fitted to change the position of the node

(d) other engines with different crank configuration could be applied to the rig (a four cylinder has been fitted).

13. REFERENCES

1. A handbook on torsional vibration
BICERA, ed. E.J.Nestorides. C.U.P. 1958.

2. Practical solution of torsional vibration problems
W.Ker Wilson. Chapman & Hall. 1963.

3. The damping capacity of engineering materials
W.H. Hatfield. Trans.N.E.C. Inst,vol.LVIII, 1942 p.273 - 332 & vol.LX, 1943-44, p.227 - 262.

4. Damping influences in torsional oscillation
J.F.Shannon. Proc.I.Mech.E. vol.131, 1935, p.387

5. Influence of engine inertia forces on minimum film thickness in con-rod big end bearings.
F.A.Martin & J.F.Booker. Proc.I.Mech.E.vol.181 Pt.1, 1966-7.

6. Crankshaft damping.
P. Draminsky. Proc.I.Mech.E. vol.159, 1948, p.416

7. Principles of lubrication.
A. Cameron. Longmans. 1966.

8. Theory of hydrodynamic lubrication.
O. Pinkus & B. Sternlicht. McGraw-Hill. 1961.

9. Elastic hysteresis in crankshaft steels. S. F. Dorey.
Proc.I.Mech.E. 1932, vol.123.

10. Mechanical vibrations
J.M.Prentis & F.A.Leckie. Longmans. 1963

11. Engineering dynamics, vol. 4
C.B.Biezeno & R. Grammel. Blackie & Son Ltd. 1954

12. Properties of torsional vibration in reciprocating engine shafts.
G.R.Goldsbrough. Roy.Soc.Proc.A. vol.109, p.99. 1926

13. Comparison of different material in vibrating structures
R.D.Adams & D.J.Mead. ISAV. Report No.158. Nov.1966.

APPENDIX 1

Print-out of 1-NODE NATURAL FREQUENCIES Ford Zephyr rig with 6 pistons

0.60 intermediate shaft

F= 2270.23461 VIB/MIN

MASS	INERTIA J	INERTIA TORQUE PER.RAD. LB.IN/RAD.	TWIST AT MASS M (RAD)	TOTAL TORQUE AT MASS M (LB.IN)	SHAFT STIFFNESS K
1	.08000000	4521.08750	1.000000000	4521.08750	8900000.0
2	.08000000	4521.08750	.999492013	9039.87838	5630000.0
3	.06880000	3888.13526	.997886350	12919.7955	9290000.0
4	.06880000	3888.13526	.996495628	16794.3052	5630000.0
5	.08000000	4521.08750	.993512624	21286.0628	8950000.0
6	.08000000	4521.08750	.991134295	25767.0676	22600.000
7	3.0600000	172931.599	-.149001444	.009887695	.00000000

0.65 intermediate shaft

F= 2665.10602 VIB/MIN

MASS	INERTIA J	INERTIA TORQUE PER.RAD. LB.IN/RAD.	TWIST AT MASS M (RAD)	TOTAL TORQUE AT MASS M (LB.IN)	SHAFT STIFFNESS K
1	.08000000	6230.60783	1.000000000	6230.60783	8900000.0
2	.08000000	6230.60783	.999299930	12456.8538	5630000.0
3	.06880000	5358.32270	.997087346	17799.5696	9290000.0
4	.06880000	5358.32270	.995171355	23132.0188	5630000.0
5	.08000000	6230.60783	.991062646	29306.9415	8950000.0
6	.08000000	6230.60783	.987788127	35461.4620	31200.000
7	3.0600000	238320.750	-.148797194	.003417969	.00000000

APPENDIX 1 (continued)

Print out of 1-NODE NATURAL FREQUENCIES Ford Zephyr rig with 6 pistons

0.70 intermediate shaft

F= 3081.46959 VIB/MIN

MASS	INERTIA J	INERTIA TORQUE PER.RAD. LB.IN/RAD.	TWIST AT MASS M (RAD)	TOTAL TORQUE AT MASS M (LB.IN)	SHAFT STIFFNESS K
1	.08000000	8329.46688	1.000000000	8329.46688	8900000.0
2	.08000000	8329.46688	.999064104	16651.1382	5630000.0
3	.06880000	7163.34150	.996106529	23786.5895	9290000.0
4	.06880000	7163.34150	.993546077	30903.6994	5630000.0
5	.08000000	8329.46688	.988056967	39133.6871	8950000.0
6	.08000000	8329.46688	.983684489	47327.2545	41800.000
7	3.0600000	318602.108	-.148546481	.032226563	.00000000

0.75 intermediate shaft

F= 3529.95520 VIB/MIN

MASS	INERTIA J	INERTIA TORQUE PER.RAD. LB.IN/RAD.	TWIST AT MASS M (RAD)	TOTAL TORQUE AT MASS M (LB.IN)	SHAFT STIFFNESS K
1	.08000000	10930.4948	1.000000000	10930.4948	8900000.0
2	.08000000	10930.4948	.998771855	21847.5653	5630000.0
3	.06880000	9400.22549	.994891295	31199.7678	9290000.0
4	.06880000	9400.22549	.991532871	40520.4005	5630000.0
5	.08000000	10930.4948	.984335640	51279.6761	8950000.0
6	.08000000	10930.4948	.978606069	61976.3245	55000.000
7	3.0600000	418091.424	-.148236195	.042724609	.00000000

Print out of 1-NODE NATURAL FREQUENCIES Ford Zephyr rig
with 6 pistons

0.80 intermediate shaft

F= 4018.03600 VIB/MIN

MASS	INERTIA J	INERTIA TORQUE PER.RAD. LB.IN/RAD.	TWIST AT MASS M (RAD)	TOTAL TORQUE AT MASS M (LB.IN)	SHAFT STIFFNESS K
1	.08000000	14162.1463	1.000000000	14162.1463	8900000.0
2	.08000000	14162.1463	.998408747	28301.7571	5630000.0
3	.06880000	12179.4458	.993381792	40400.5968	9290000.0
4	.06880000	12179.4458	.989032967	52446.4703	5630000.0
5	.08000000	14162.1463	.979717431	66321.3718	8950000.0
6	.08000000	14162.1463	.972307221	80091.3288	71500.000
7	3.0600000	541702.095	-.147851223	.011230469	.00000000

Print out of 1-NODE NATURAL FREQUENCIES Ford Zephyr rig
with only No.1 piston

0.60 intermediate shaft

F= 2418.74207 VIB/MIN

MASS	INERTIA J	INERTIA TORQUE PER.RAD. LB.IN/RAD.	TWIST AT MASS M (RAD)	TOTAL TORQUE AT MASS M (LB.IN)	SHAFT STIFFNESS K
1	.08000000	5131.92796	1.000000000	5131.92796	8900000.0
2	.06780000	4349.30894	.999423379	9478.72903	5630000.0
3	.05630000	3611.59429	.997739767	13082.1603	9290000.0
4	.05630000	3611.59429	.996331570	16680.5056	5630000.0
5	.06780000	4349.30894	.993368780	21000.9734	8950000.0
6	.06780000	4349.30894	.991022301	25311.2356	22600.000
7	3.0600000	196296.244	-.128943881	.036132813	.00000000

APPENDIX 1 (Continued)

Print out of 1-NODE NATURAL FREQUENCIES Ford Zephyr rig

with No.1 piston only

0.65 intermediate shaft

F= 2839.34409 VIB/MIN

MASS	INERTIA J	INERTIA TORQUE PER.RAD. LB.IN/RAD.	TWIST AT MASS M (RAD)	TOTAL TORQUE AT MASS M (LB.IN)	SHAFT STIFFNESS K
1	.08000000	7071.92234	1.000000000	7071.92234	8900000.0
2	.06780000	5993.45420	.999205401	13060.6142	5630000.0
3	.05630000	4976.86537	.996885577	18021.9794	9290000.0
4	.05630000	4976.86537	.994945643	22973.6900	5630000.0
5	.06780000	5993.45420	.990865057	28912.3943	8950000.0
6	.06780000	5993.45420	.987634623	34831.7371	31200.000
7	3.0600000	270501.030	-.128767209	.074462891	.00000000

0.70 intermediate shaft

F= 3282.78656 VIB/MIN

MASS	INERTIA J	INERTIA TORQUE PER.RAD. LB.IN/RAD.	TWIST AT MASS M (RAD)	TOTAL TORQUE AT MASS M (LB.IN)	SHAFT STIFFNESS K
1	.08000000	9453.37155	1.000000000	9453.37155	8900000.0
2	.06780000	8011.73243	.998937824	17456.5941	5630000.0
3	.05630000	6652.81025	.995837187	24081.7100	9290000.0
4	.05630000	6652.81025	.993244970	30689.5803	5630000.0
5	.06780000	8011.73243	.987793890	38603.5206	8950000.0
6	.06780000	8011.73243	.983480649	46482.9044	41800.000
7	3.0600000	361591.461	-.128550554	.121826172	.00000000

APPENDIX 1 (Continued)

Print out of 1-NODE NATURAL FREQUENCIES Ford Zephyr rig
with No.1 piston only

0.75 intermediate shaft

F= 3760.37055 VIB/MIN

MASS	INERTIA J	INERTIA TORQUE PER.RAD. LB.IN/RAD.	TWIST AT MASS M (RAD)	TOTAL TORQUE AT MASS M (LB.IN)	SHAFT STIFFNESS K
1	.08000000	12404.0275	1.000000000	12404.0275	8900000.0
2	.06780000	10512.4134	.998606288	22901.7896	5630000.0
3	.05630000	8729.33438	.994538476	31583.4485	9290000.0
4	.05630000	8729.33438	.991138750	40235.4300	5630000.0
5	.06780000	10512.4134	.983992138	50579.5621	8950000.0
6	.06780000	10512.4134	.978340791	60864.2850	55000.000
7	3.0600000	474454.051	-.128282571	.099365235	.00000000

0.80 intermediate shaft

F= 4280.02388 VIB/MIN

MASS	INERTIA J	INERTIA TORQUE PER.RAD. LB.IN/RAD.	TWIST AT MASS M (RAD)	TOTAL TORQUE AT MASS M (LB.IN)	SHAFT STIFFNESS K
1	.08000000	16069.1838	1.000000000	16069.1838	8900000.0
2	.06780000	13618.6333	.998194472	29663.2282	5630000.0
3	.05630000	11308.6881	.992925694	40891.9151	9290000.0
4	.05630000	11308.6881	.988523980	52070.8245	5630000.0
5	.06780000	13618.6333	.979275165	65407.2139	8950000.0
6	.06780000	13618.6333	.971967095	78644.0773	71500.000
7	3.0600000	614646.279	-.127950074	.040527344	.00000000

APPENDIX 2

Relationships of the damping terms at resonance.

The logarithmic decrement $\delta = \log_e \dfrac{\theta_n}{\theta_{n+1}}$

Damping ratio $\zeta = \dfrac{c}{c_c}$ is defined in terms of the single degree of freedom system where C_c = critical damping.

The effect of damping ratio on the natural frequency is seen in the expression.

$$\omega_d = \sqrt{1 - 2(\zeta)^2}$$

where ω_d = damped frequency of resonance.

ω_n = natural (undamped) frequency of resonance,

and $\delta = \dfrac{2\pi\zeta}{\sqrt{1-\zeta^2}} \doteq 2\pi\zeta$ if ζ is small.

$$\frac{\Delta W}{W} = \frac{\frac{1}{2}k(\theta_n^2 - \theta_{n+1}^2)}{\frac{1}{2}k(\theta_n^2)} = 1 - \frac{1}{e^{2\delta}}$$

$$= 2\delta\left(1 - \delta + \frac{2}{3}\delta^2 - \frac{1}{3}\delta^3 + \cdots\right)$$

$\doteq 2\delta$ if δ is sufficiently small

$$M = \frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2}}$$

The maximum value of M occurs when $\omega = \omega_n\sqrt{1-2(\zeta)^2}$

$$M_{max} = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

and the phase angle ϕ by which the excitation torque leads the displacement is given by

$$\tan\phi = \frac{2\zeta\frac{\omega}{\omega_n}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

Consider the torsional pendulum:-

The externally applied harmonic torque $= \pm Q$

The twist amplitude of the shaft $= \pm \theta_t$

At resonance the input energy is entirely absorbed by the damping $\Delta W = \pi Q \theta_t$

The strain energy of the shaft $W = \dfrac{k\theta_t^2}{2}$

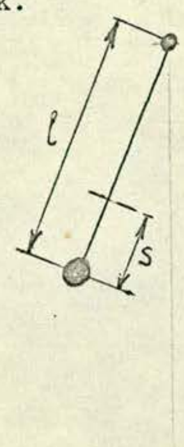
$$\text{hence} \quad \frac{\Delta W}{W} = \frac{2\pi Q}{k\,\theta_t}$$

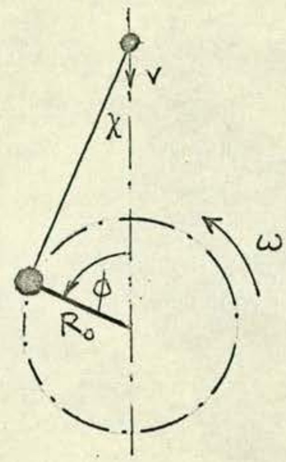
but $\dfrac{Q}{K} = \theta_{ot}$ the equilibrium amplitude of the twist

$$\therefore \quad \frac{\Delta W}{W} = \frac{2\pi}{M}$$

APPENDIX 3

The inertia effects of the connecting rod, piston and crank.





Let W = weight of revolving crank (equivalent at radius R_o)

W' = " " connecting rod

W'' = " " piston etc.

K = radius of gyration of connecting rod about C.G.

S = distance of C.G. of connecting rod from big end

R_o = crank radius

l = length of connecting rod

$\lambda = \frac{R}{l} = \frac{1}{n}$

The effective moment of inertia of the assembly is

$$I_e = R + R'\left(\frac{v}{\omega R}\right)^2 - R''\left(\frac{\dot{\chi}}{\lambda\omega}\right)^2$$

where $R = \frac{1}{g}\left[W R_o^2 + R_o^2\left(1 - \frac{s}{l}\right)W'\right]$

$R' = \frac{R_o^2}{g}\left[\frac{s}{l}W' + W''\right]$

$R'' = \frac{\lambda^2}{g}\left[s(l-s) - k^2\right]W'$ Reference 11 p.87

but $\left(\frac{V}{\omega R}\right)^2 = B_0' + B_1' \cos\phi - B_2' \cos 2\phi - B_3' \cos 3\phi - - - -$

and $\frac{\dot{X}}{\lambda\omega} = D_0' + D_2' \cos 2\phi - D_4' \cos 4\phi + - - - -$

where the constants are evaluated in reference 11, p.89.

$B_0' = \frac{1}{2} + \frac{1}{2}(.15)^2 = 0.511$ approximately

$B_1' = 0.15$

The customary approximation is to take B_0' as $\frac{1}{2}$ and to ignore B_1' etc.

$D_0' = \frac{1}{2}\left((1.01)^2 + (0.011)^2 + -----\right) = 0.51$ approximately

$D_2' = \frac{1}{2}(1.01)^2 - (1.01)(0.011) = 0.50$ approximately

$D_4' =$ etc are very small.

The term $- R''\left(\frac{\dot{X}}{\lambda\omega}\right)^2$ in the value of I_e is best described as the connecting rod inertia couple term. It has no 3rd order but the 2nd order can be appreciable, and will be evaluated for the single piston tests where a 2nd order oscillation was recorded.

A connecting rod from the engine was oscillated about both bearings and the radius of gyration K determined at 2.42 in. The result is a value for $S(l - s) - K^2$ of

$$(4.1825 \times 1.13) - (2.42)^2 = -1.12$$

leading to a positive correction to the 2nd order inertia torque of

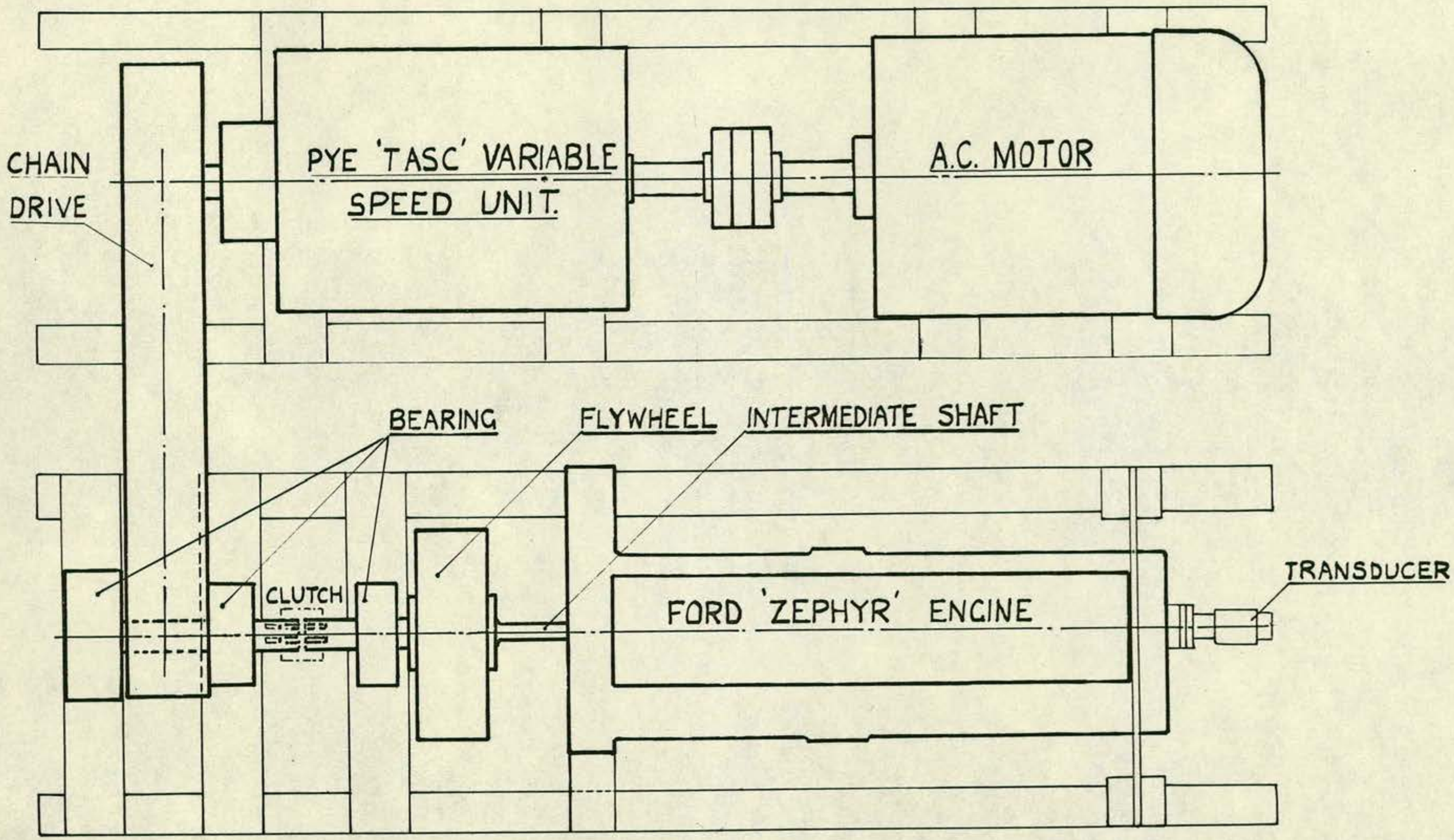
$$Q_c = -\frac{1}{2}\,\frac{-1.12}{386}\left[\frac{1.469}{3.4^2}\right]\omega^2 = 0.000184\,\omega^2 \text{ lbf.in.}$$

As the 2nd order inertia torque is negative this leads to a reduction in the exciting torque.

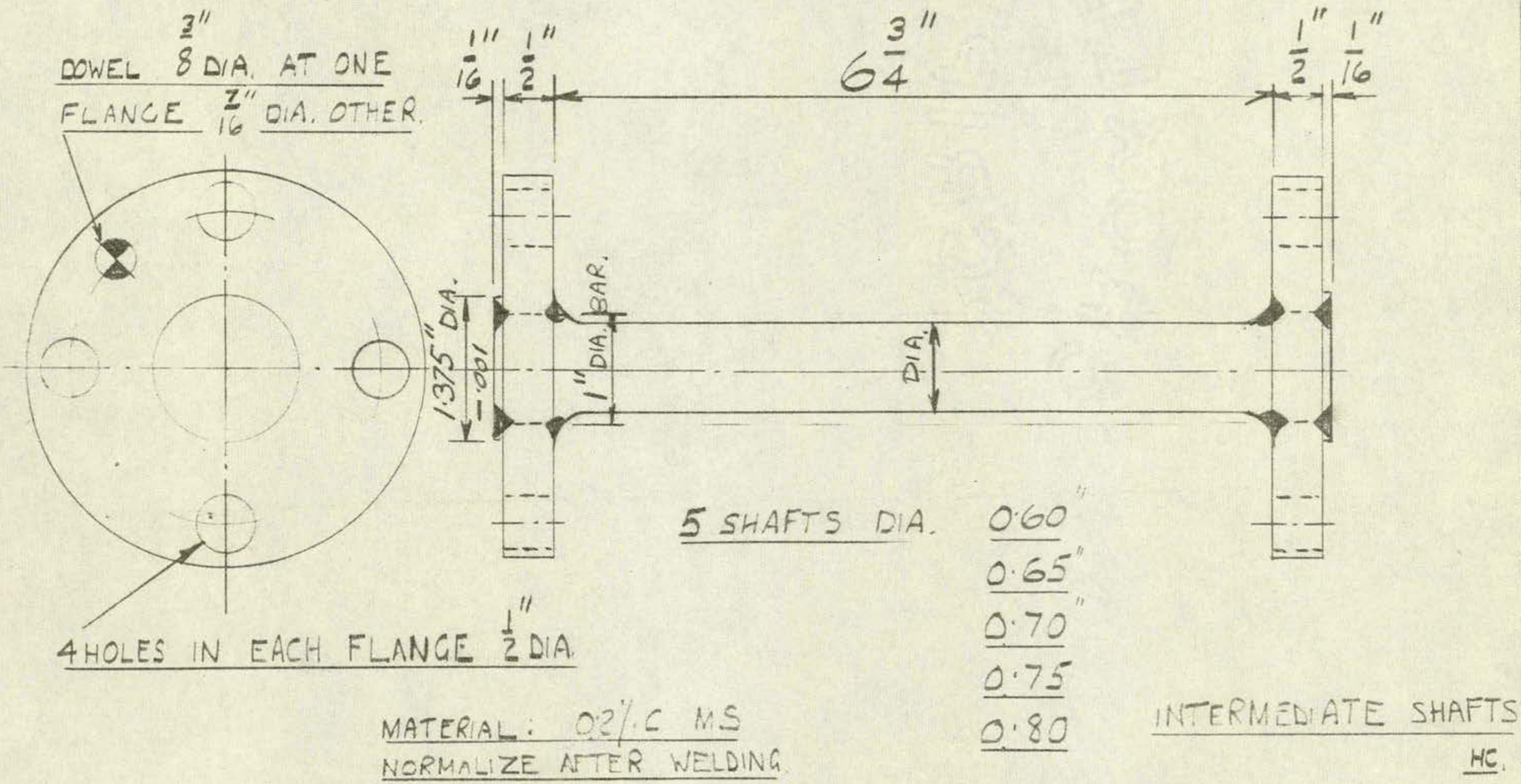
The correction should be applied to table 3

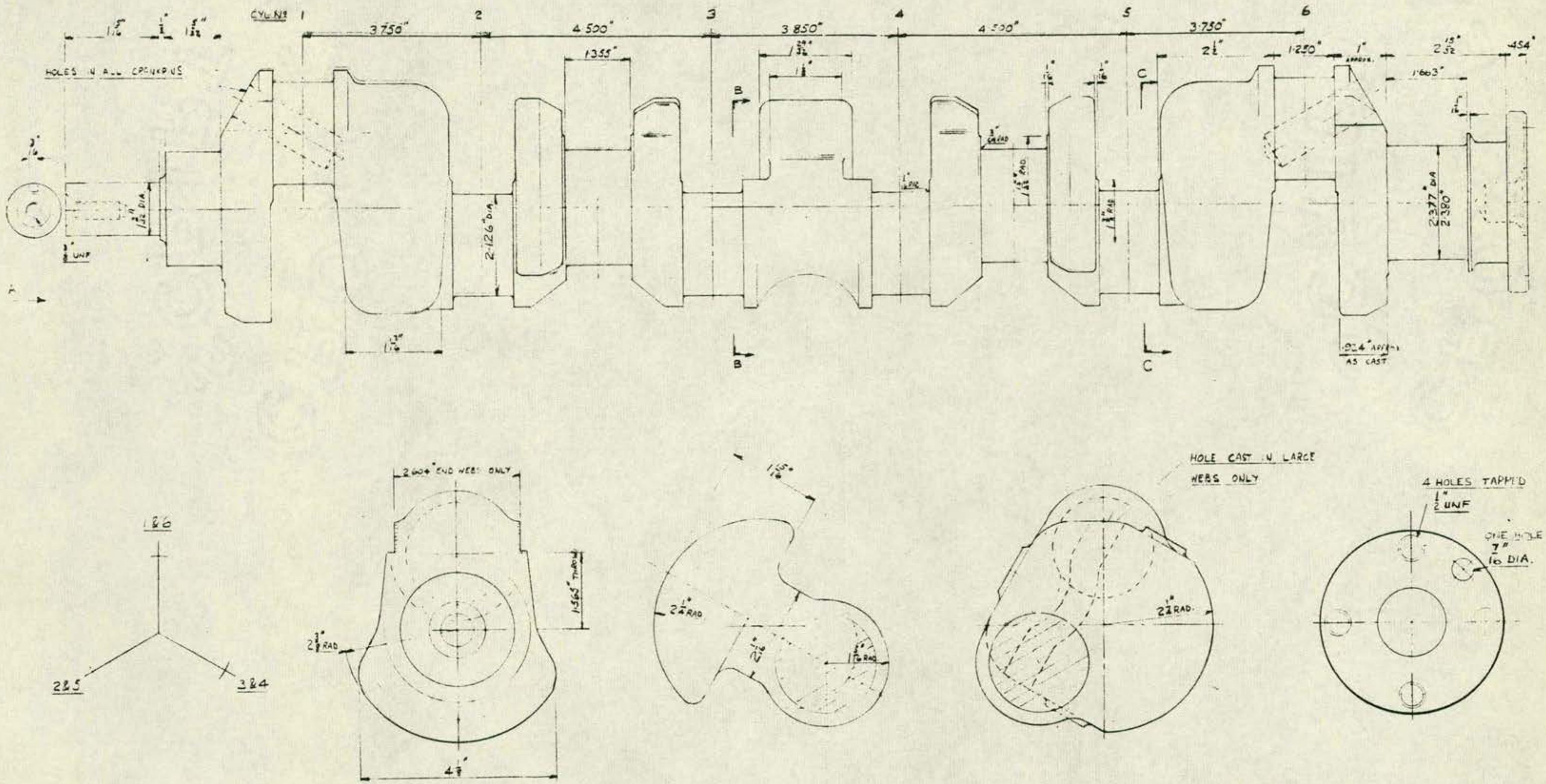
intermediate shaft	ω_f^2	Q_c	Corrected $\sum Q_a$	% change
0.60	16,380	3.02	50.5	5.5
0.65	21,600	3.98	66.6	"
0.70	27,900	5.13	86.1	"
0.75	37,800	6.97	116.6	"
0.80	44,500	8.20	137.4	"

TABLE 22



PLAN ARRANGEMENT OF THE RIG. FIG. 1


DOWEL 3/8" DIA. AT ONE
FLANGE 7/16" DIA. OTHER.
1/16"
1/2"
6 3/4"
1/2"
1/16"
1.375" DIA. −.001
1" DIA. BAR.
DIA.
4 HOLES IN EACH FLANGE 1/2" DIA.
5 SHAFTS DIA.
0·60"
0·65"
0·70"
0·75
0·80
MATERIAL: 0·2% C MS
NORMALIZE AFTER WELDING
INTERMEDIATE SHAFTS
HC.



VIEW AS ARROW 'A'

VIEW AS ARROW 'A'

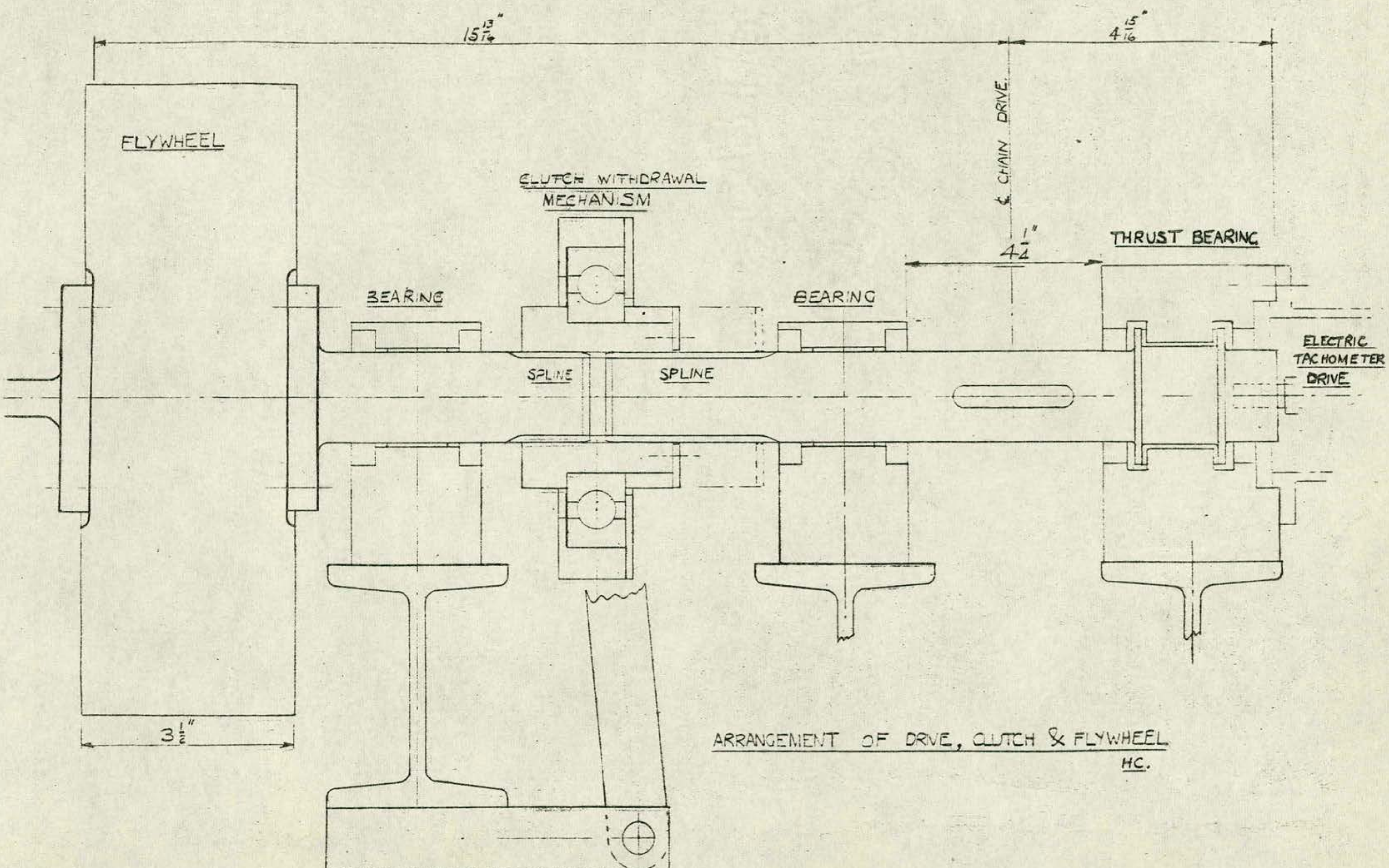
SECTION 'BB'

SECTION CC

CRANKSHAFT

FORD ZEPHYR HC

'G' FOR MATL. = 11.35×10^6 LBF/SQ IN.



ARRANGEMENT OF DRIVE, CLUTCH & FLYWHEEL

HC.

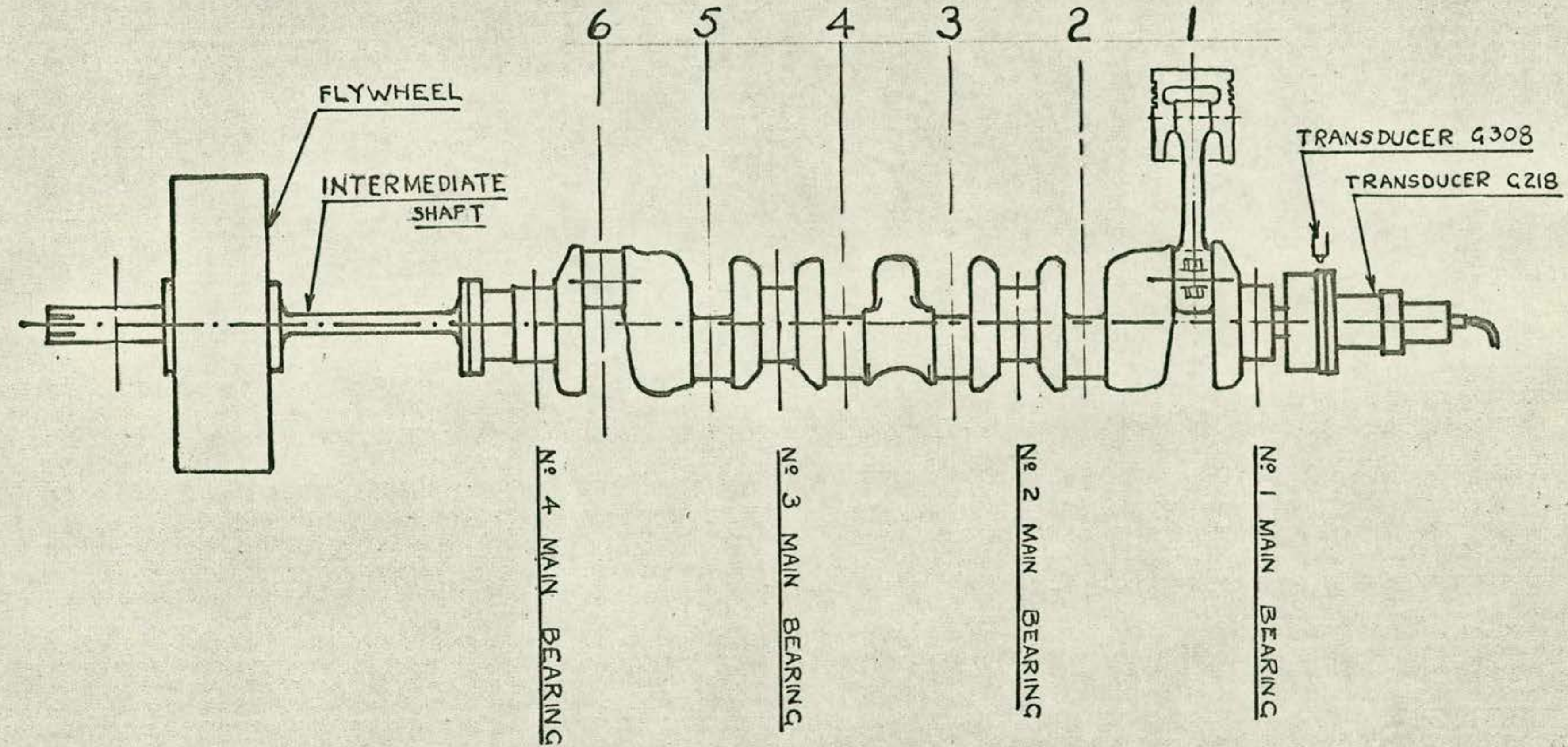


FIG. 5. THE ESSENTIALS OF THE TORSIONAL SYSTEM.

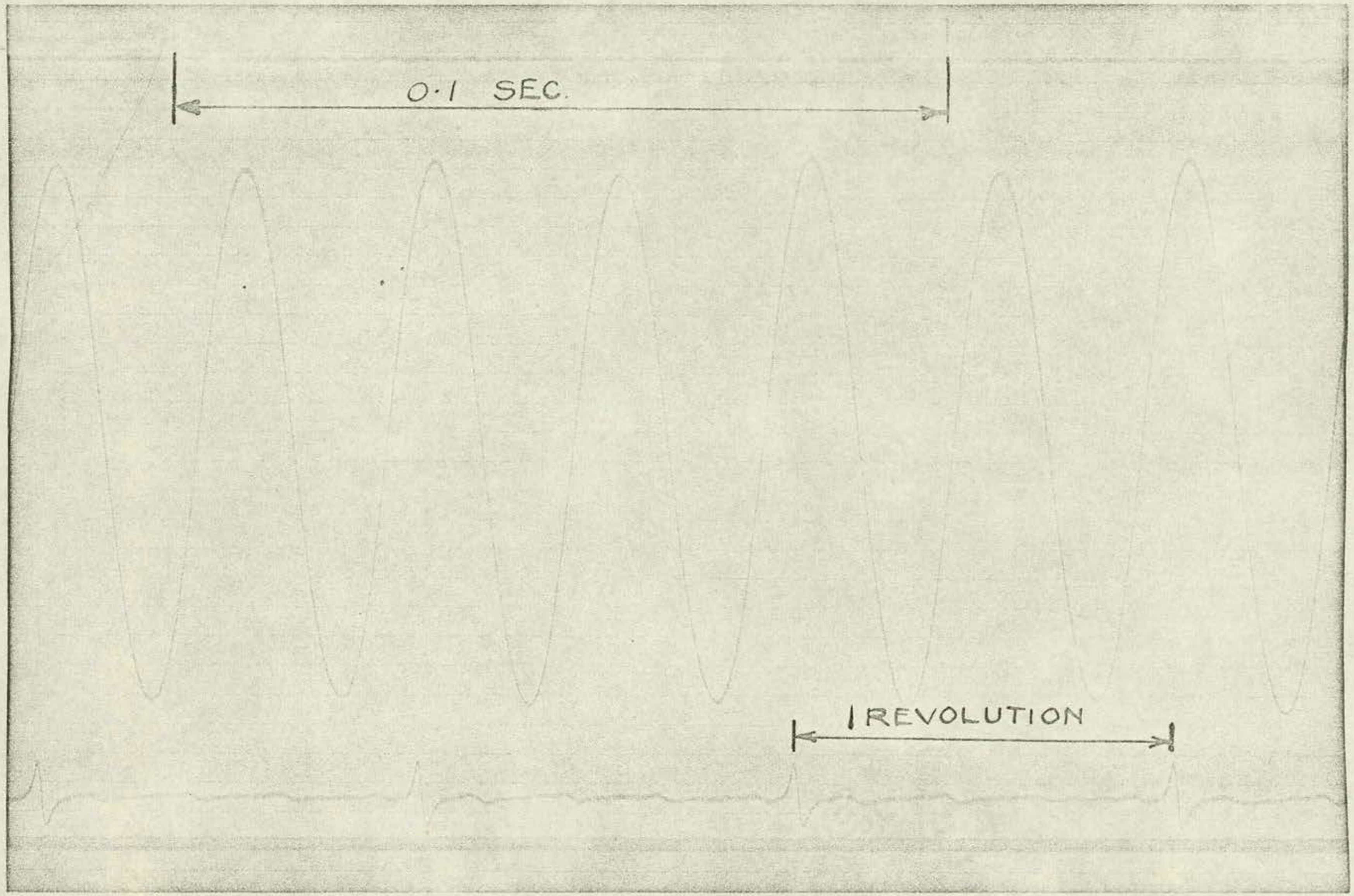


Fig.No.6 Typical record, 2nd order oscillation 0.60" shaft and No.1 piston only, with three piston rings.

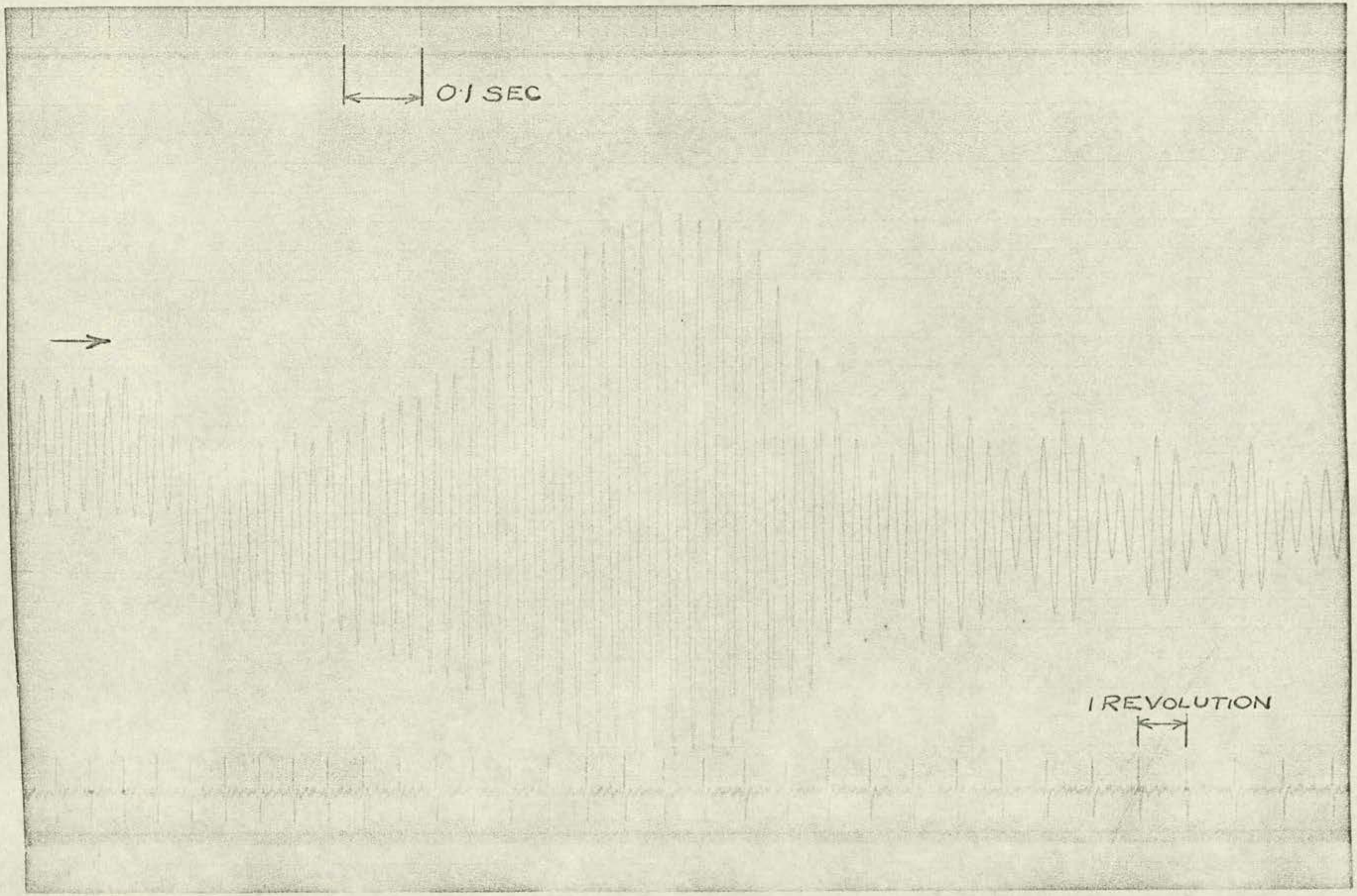


Fig No.7   Run down test, 2nd order oscillation 0.60" shaft and No.1 piston only with three piston rings.

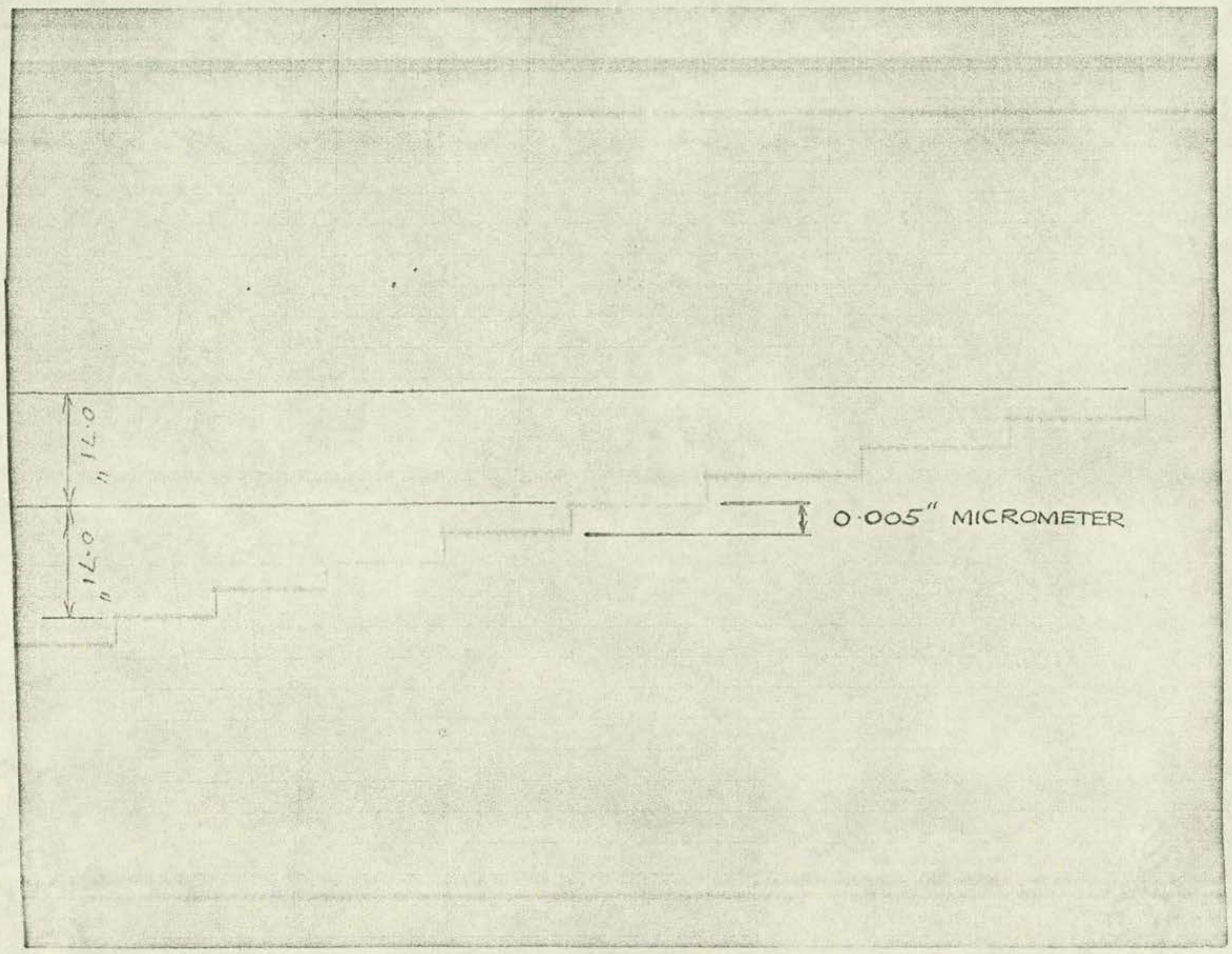


Fig.No.8 Typical calibration record.

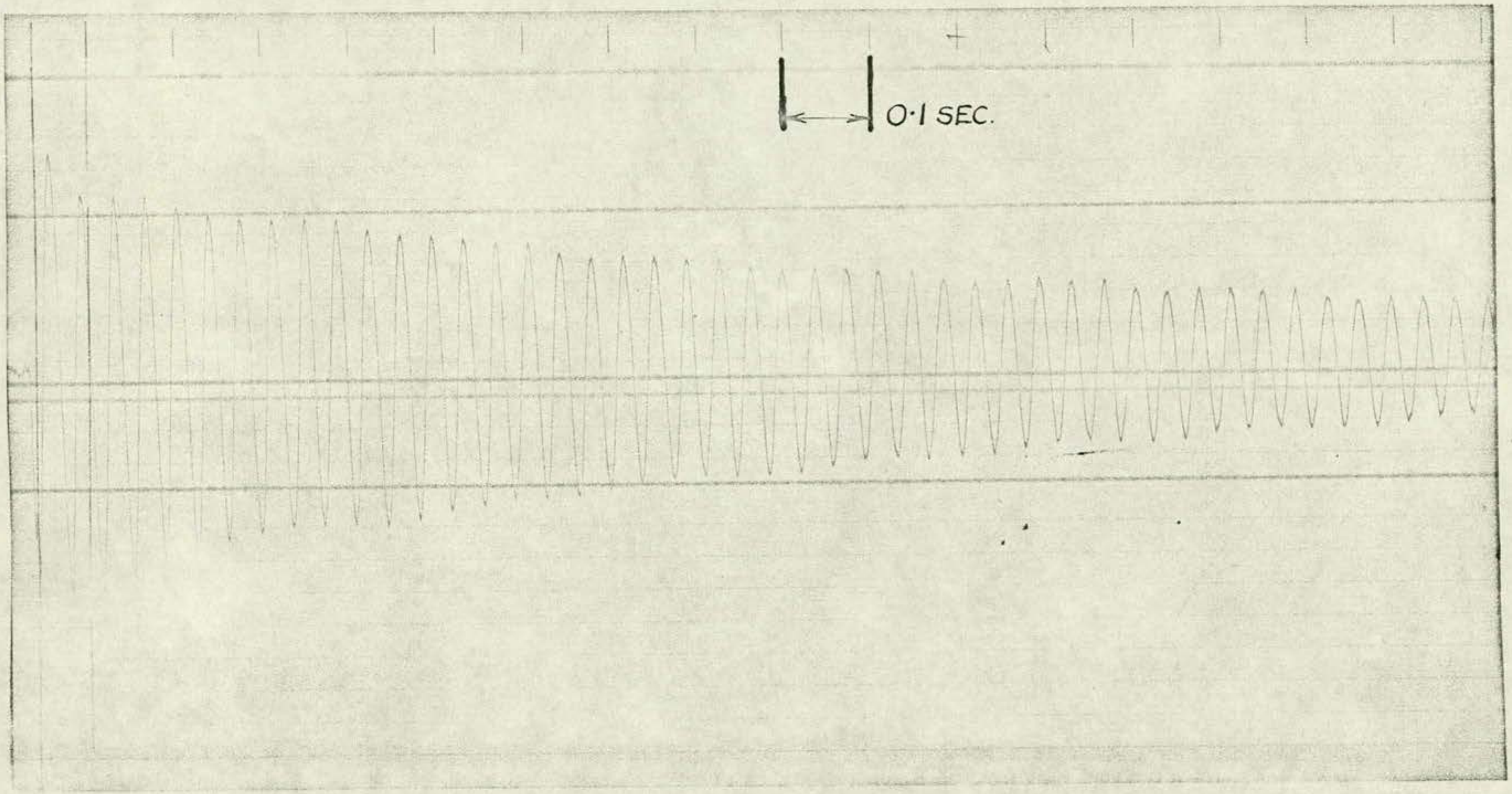


Fig. No.9 Typical log decrement curve, 0.375" waisted shaft.

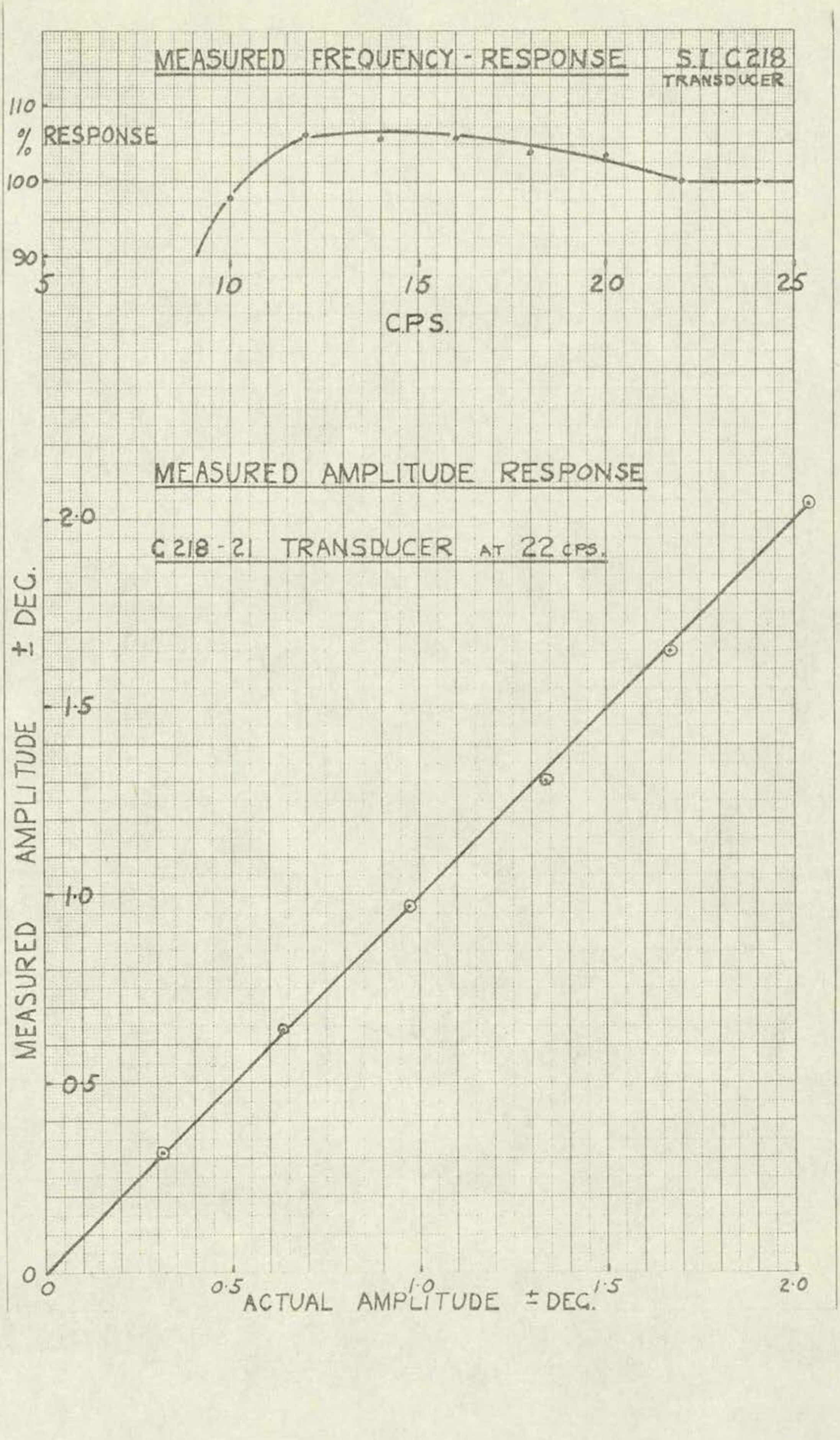

MEASURED FREQUENCY - RESPONSE
S.I. C218 TRANSDUCER
% RESPONSE
110
100
90
5
10
15
20
25
C.P.S.
MEASURED AMPLITUDE RESPONSE
C218-21 TRANSDUCER AT 22 CPS.
MEASURED AMPLITUDE ± DEG.
2·0
1·5
1·0
0·5
0
0
0·5
1·0
1·5
2·0
ACTUAL AMPLITUDE ± DEG.

FIG. 11

MEASURED DAMPING OF INTERMEDIATE SHAFTS

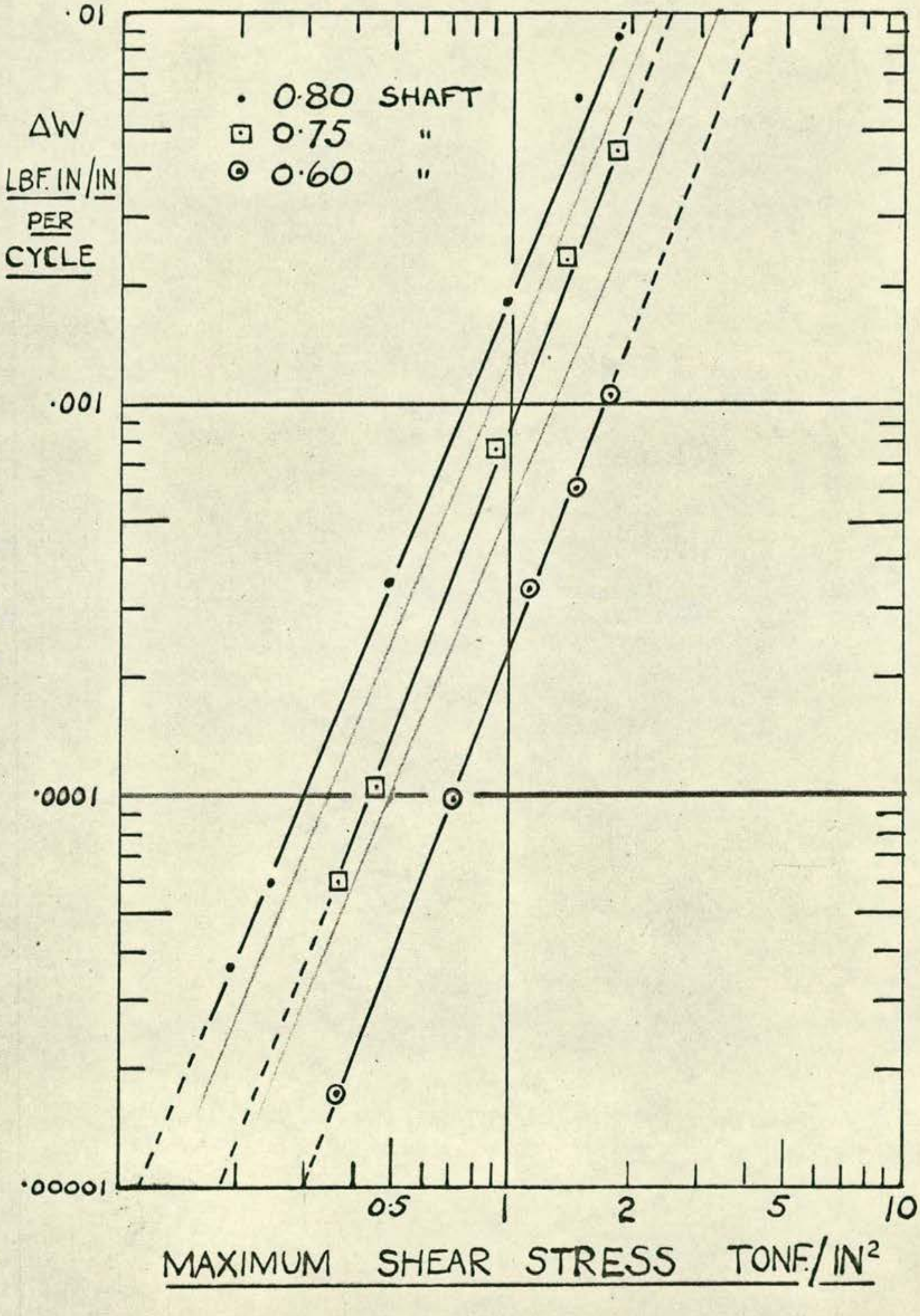
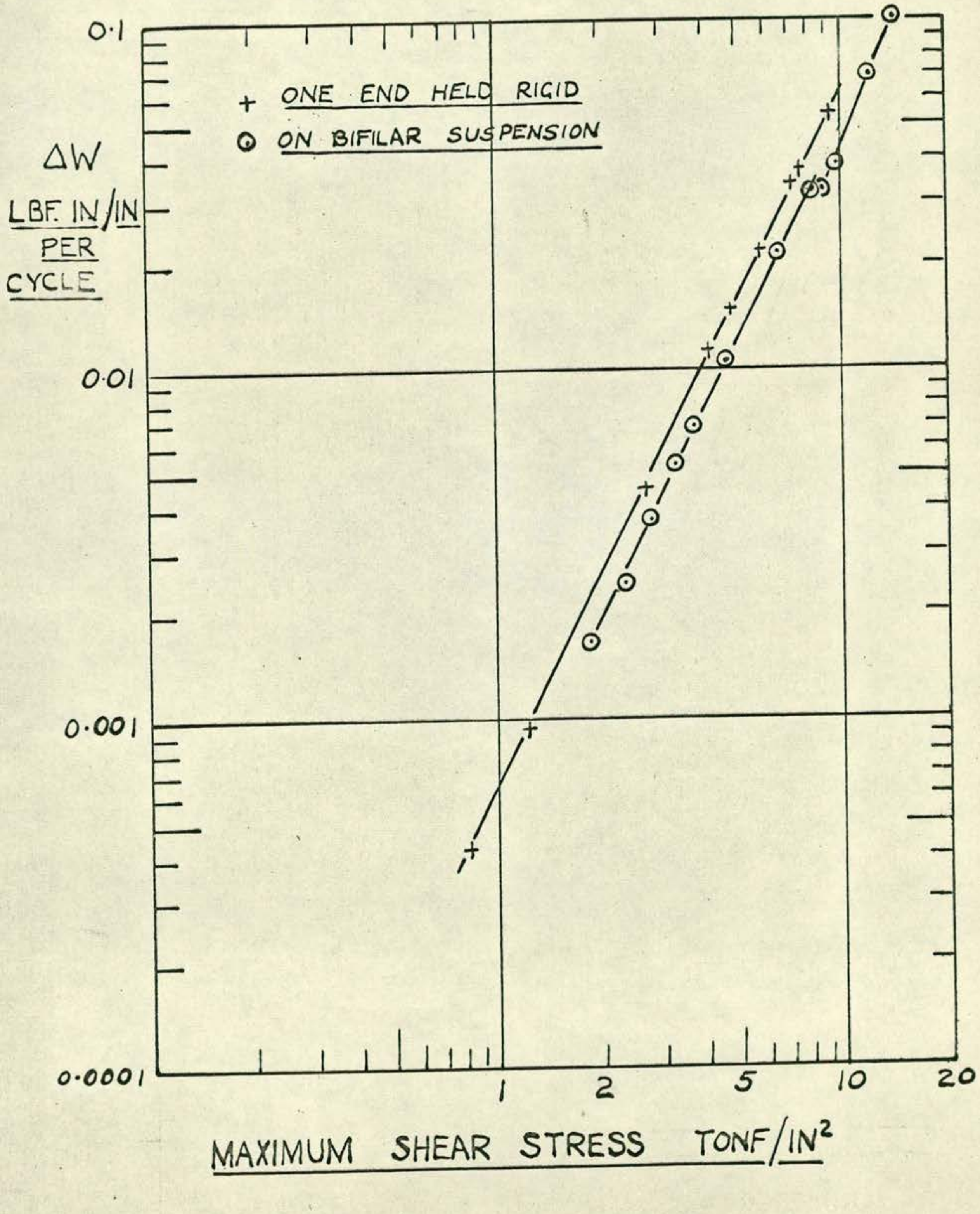


FIG. 12

DAMPING ENERGY LOSS $\frac{3}{8}''$ DIA. SPECIMEN 0·2% C M.S.



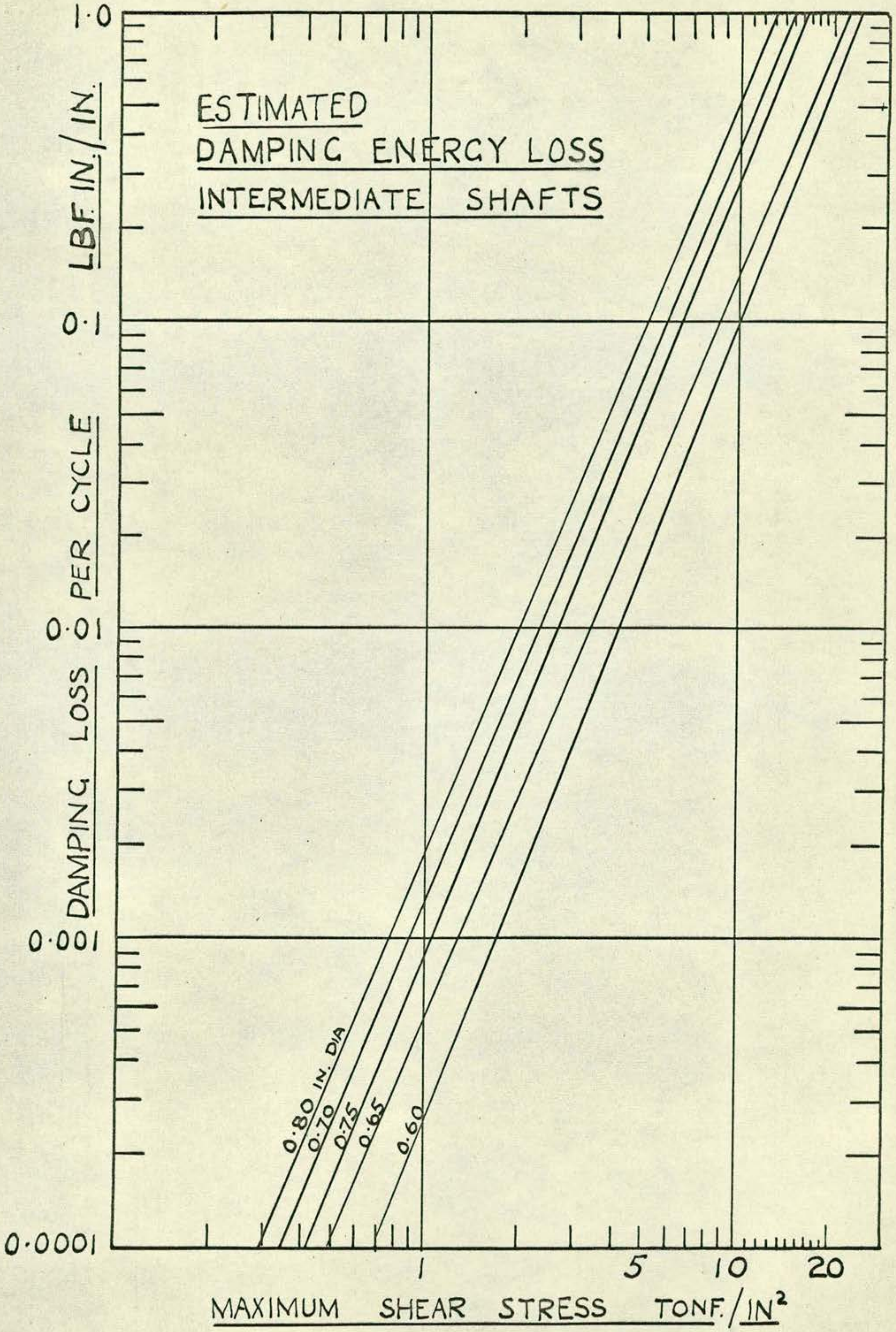
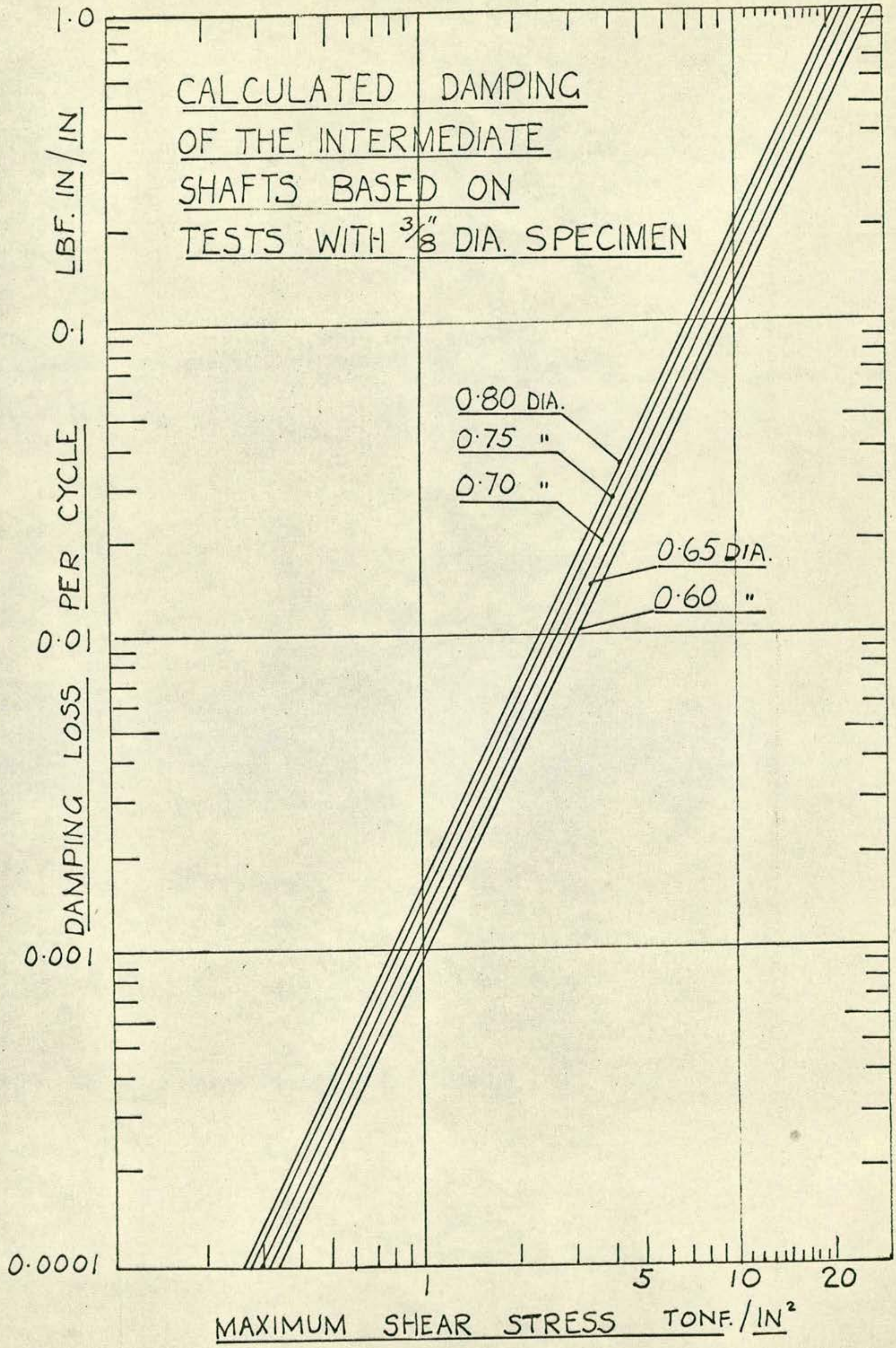
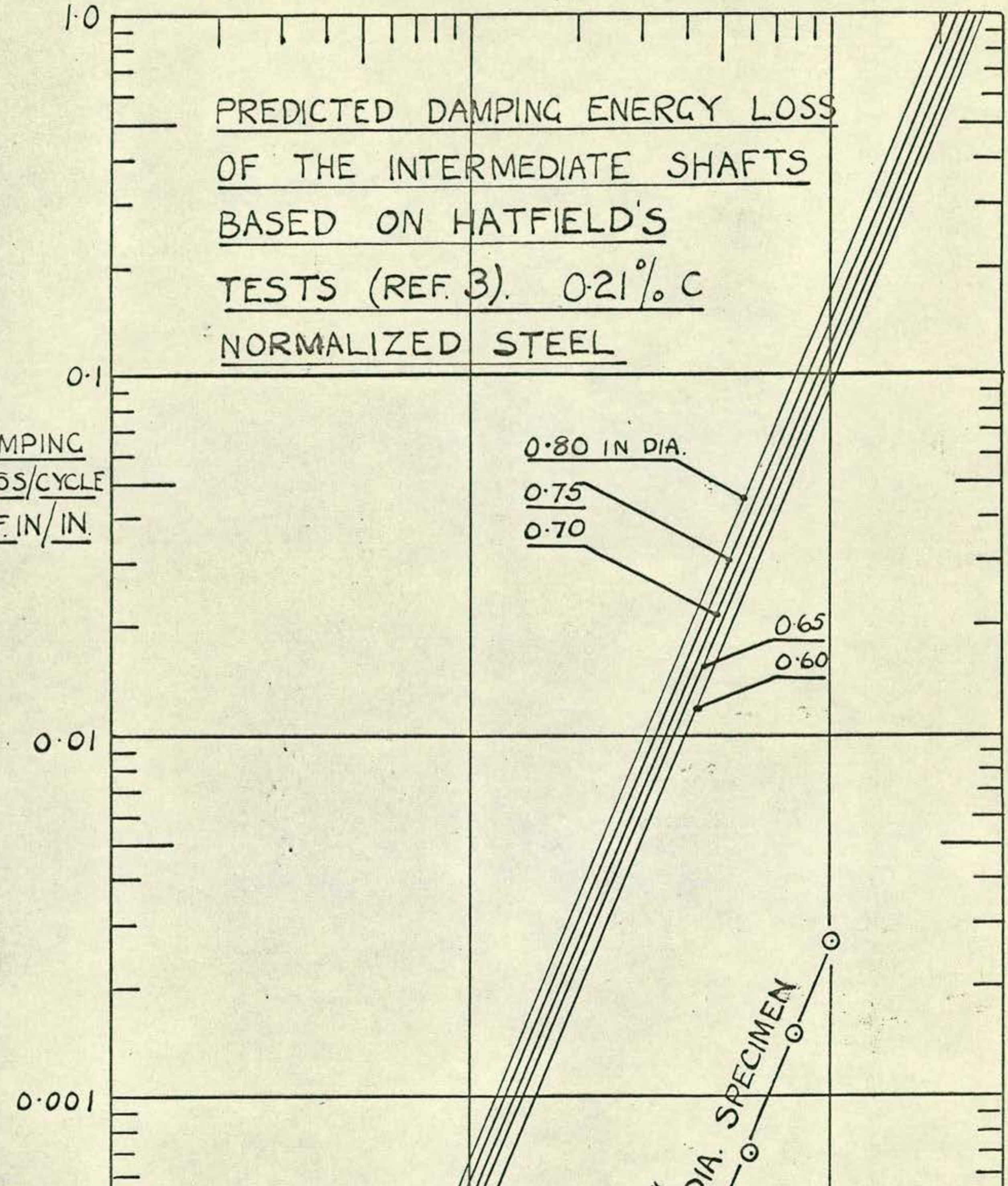

ESTIMATED
DAMPING ENERGY LOSS
INTERMEDIATE SHAFTS
DAMPING LOSS PER CYCLE LBF IN./IN.
1·0
0·1
0·01
0·001
0·0001
0·80 IN. DIA
0·70
0·75
0·65
0·60
1
5
10
20
MAXIMUM SHEAR STRESS TONF./IN²

FIG. 13



CALCULATED DAMPING OF THE INTERMEDIATE SHAFTS BASED ON TESTS WITH 3/8" DIA. SPECIMEN
LBF. IN/IN
DAMPING LOSS PER CYCLE
1.0
0.1
0.01
0.001
0.0001
0.80 DIA.
0.75 "
0.70 "
0.65 DIA.
0.60 "
1
5
10
20
MAXIMUM SHEAR STRESS TONF./IN²

FIG. 14.



PREDICTED DAMPING ENERGY LOSS
OF THE INTERMEDIATE SHAFTS
BASED ON HATFIELD'S
TESTS (REF. 3). 0·21% C
NORMALIZED STEEL
1·0
0·1
0·01
0·001
0·0001
DAMPING
LOSS/CYCLE
LBF. IN/IN.
0·80 IN DIA.
0·75
0·70
0·65
0·60
HATFIELD'S 5/16" DIA. SPECIMEN
1
2
5
10
20
MAXIMUM SHEAR STRESS TONF./IN²

FIG. 15

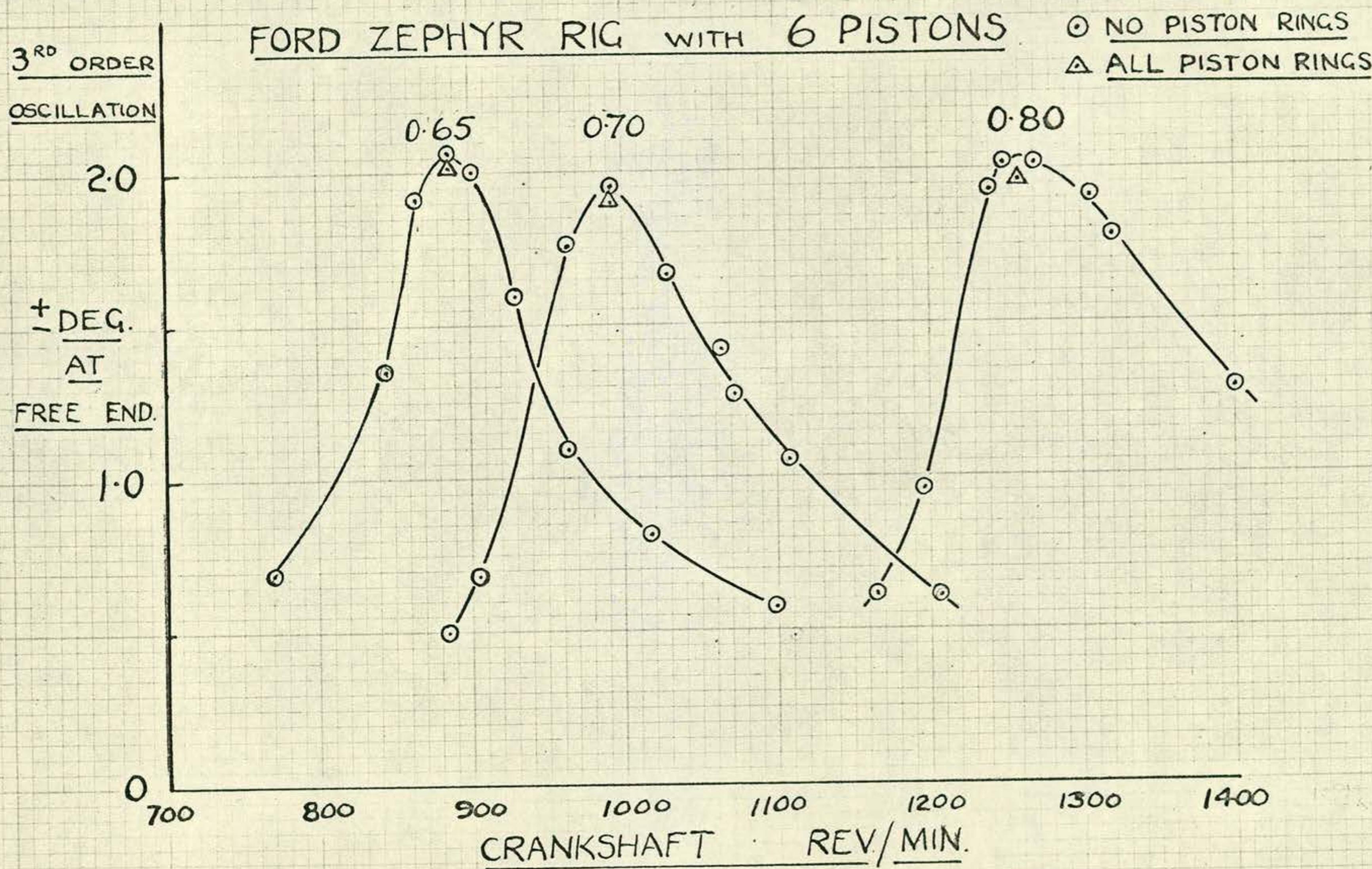
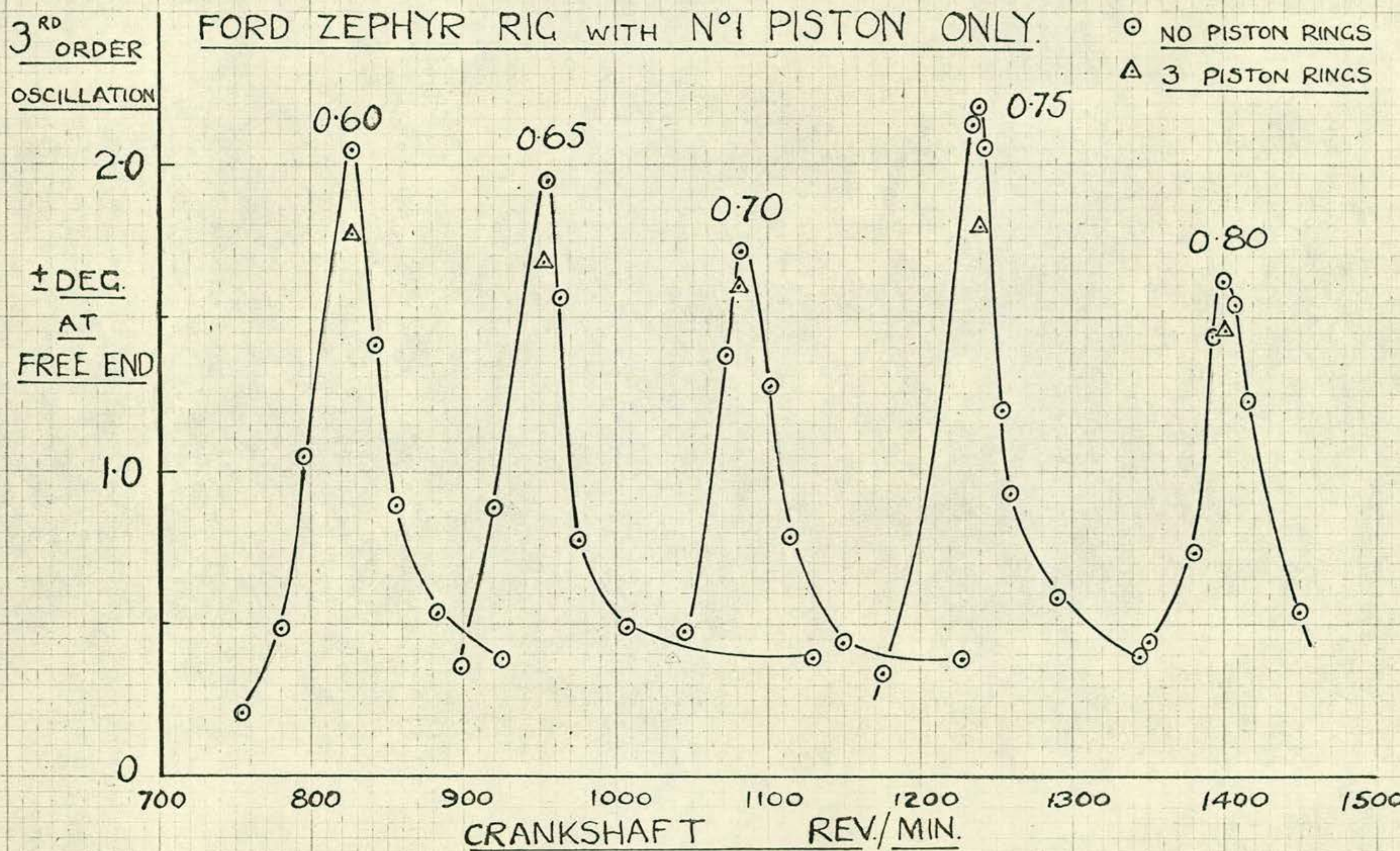

FORD ZEPHYR RIG WITH 6 PISTONS
⊙ NO PISTON RINGS
△ ALL PISTON RINGS
3RD ORDER OSCILLATION
± DEG. AT FREE END.
0·65
0·70
0·80
2·0
1·0
0
700
800
900
1000
1100
1200
1300
1400
CRANKSHAFT REV/MIN.

FIG 16.



FORD ZEPHYR RIG WITH N°1 PISTON ONLY.
NO PISTON RINGS
3 PISTON RINGS
3RD ORDER OSCILLATION
± DEG. AT FREE END
2·0
1·0
0
0·60
0·65
0·70
0·75
0·80
700
800
900
1000
1100
1200
1300
1400
1500
CRANKSHAFT REV./MIN.

FIG 17

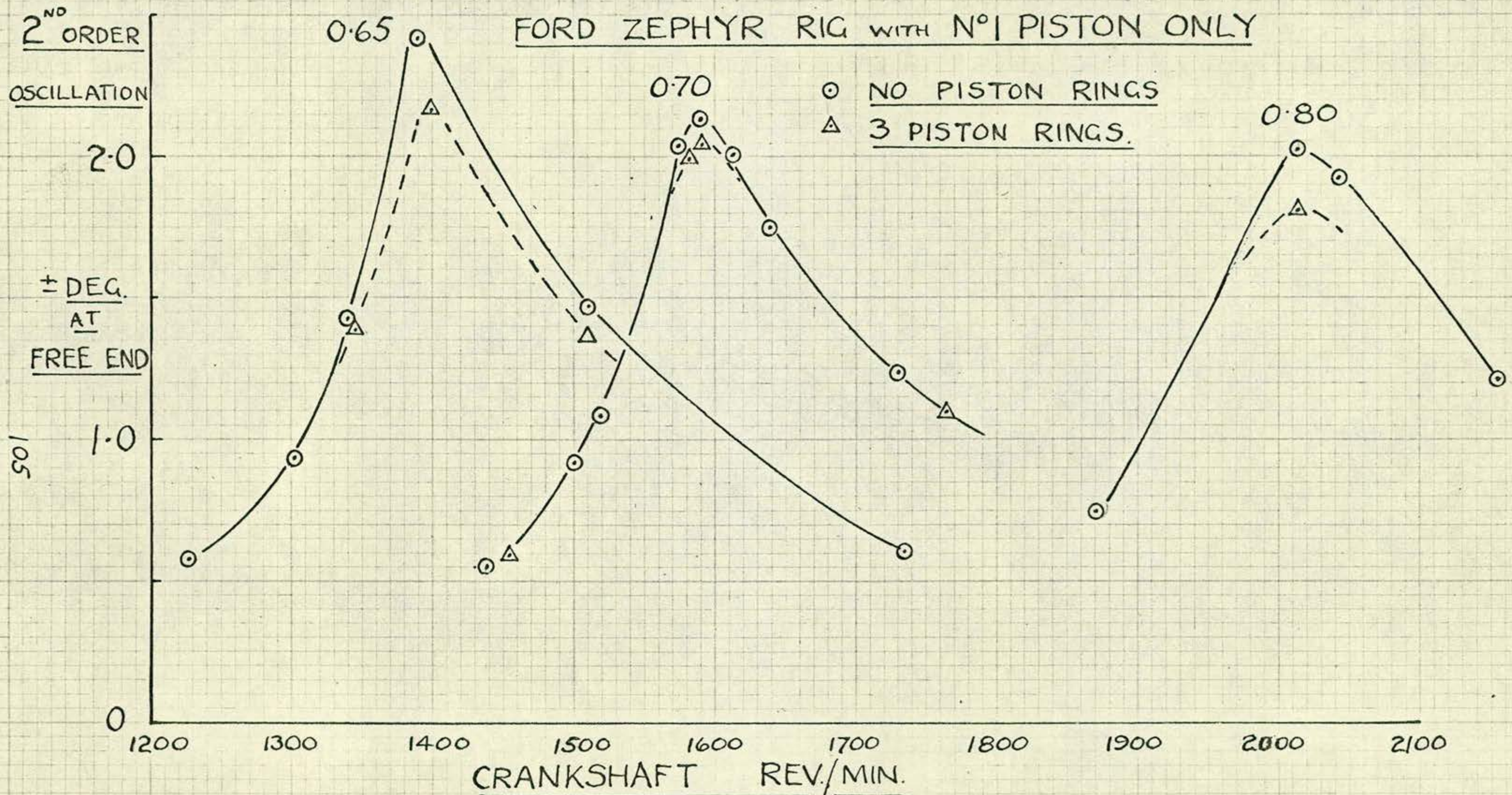
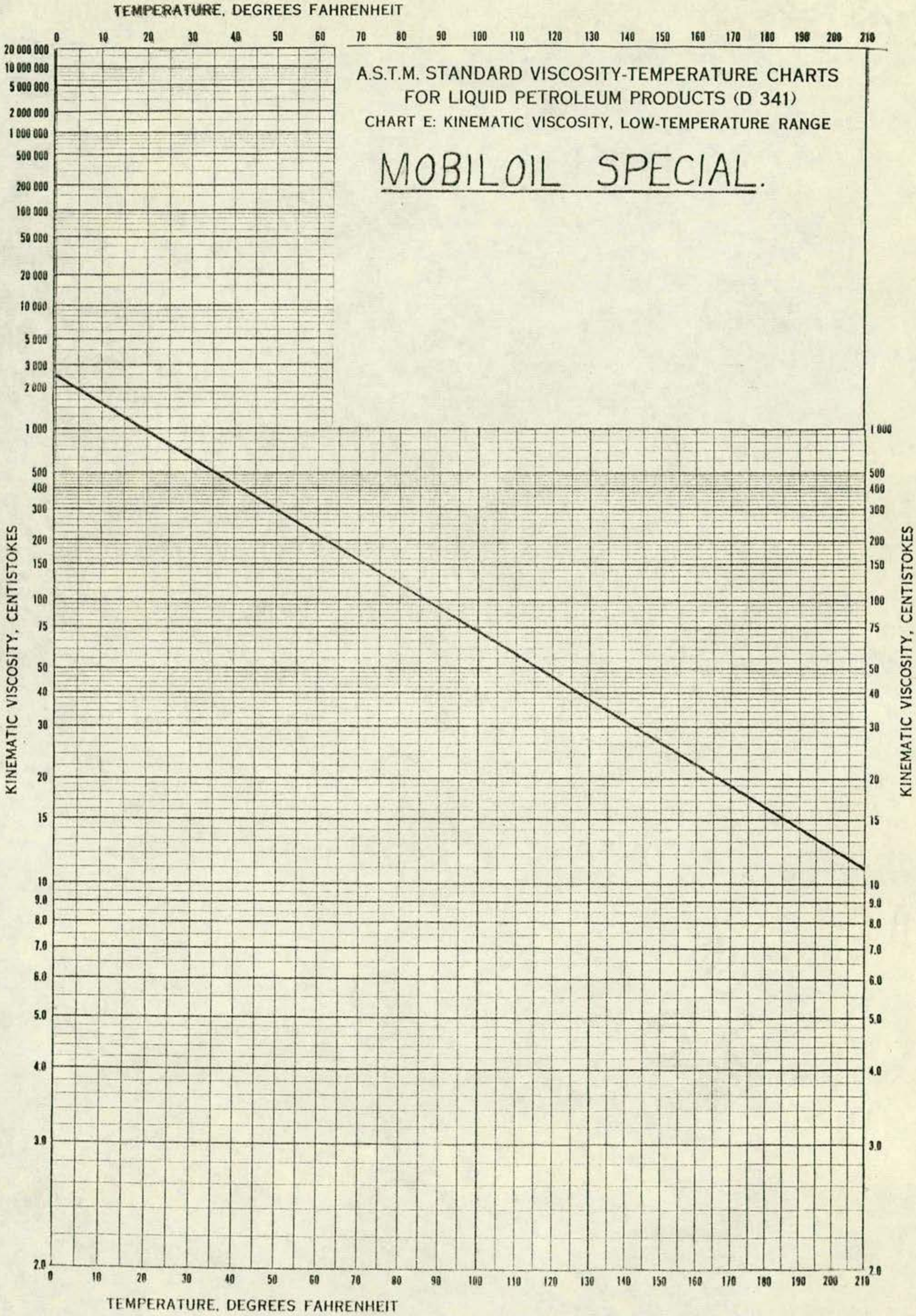
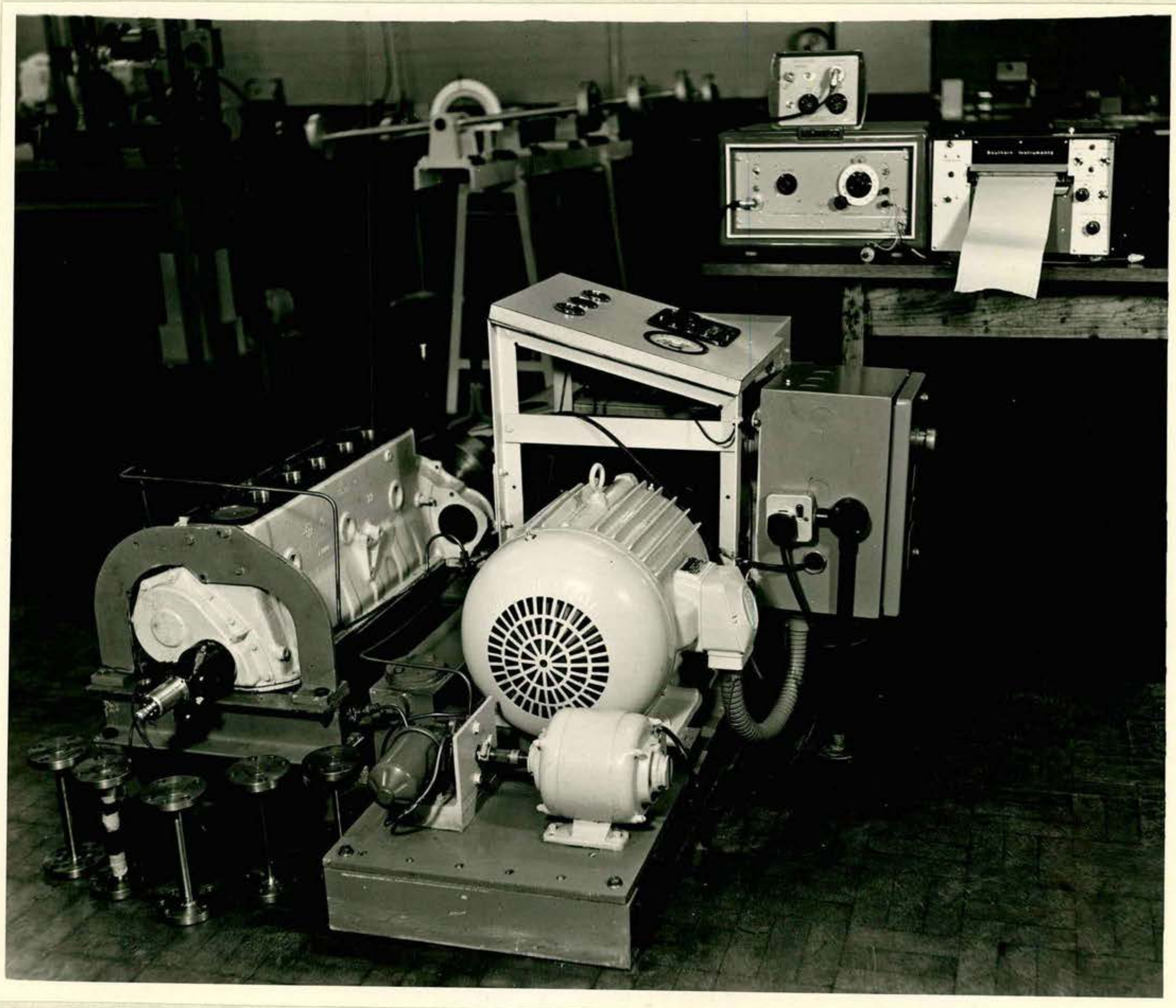

FORD ZEPHYR RIG WITH N°1 PISTON ONLY
2ND ORDER OSCILLATION
± DEG. AT FREE END
0·65
0·70
0·80
NO PISTON RINGS
3 PISTON RINGS.
2·0
1·0
0
1200
1300
1400
1500
1600
1700
1800
1900
2000
2100
CRANKSHAFT REV./MIN.

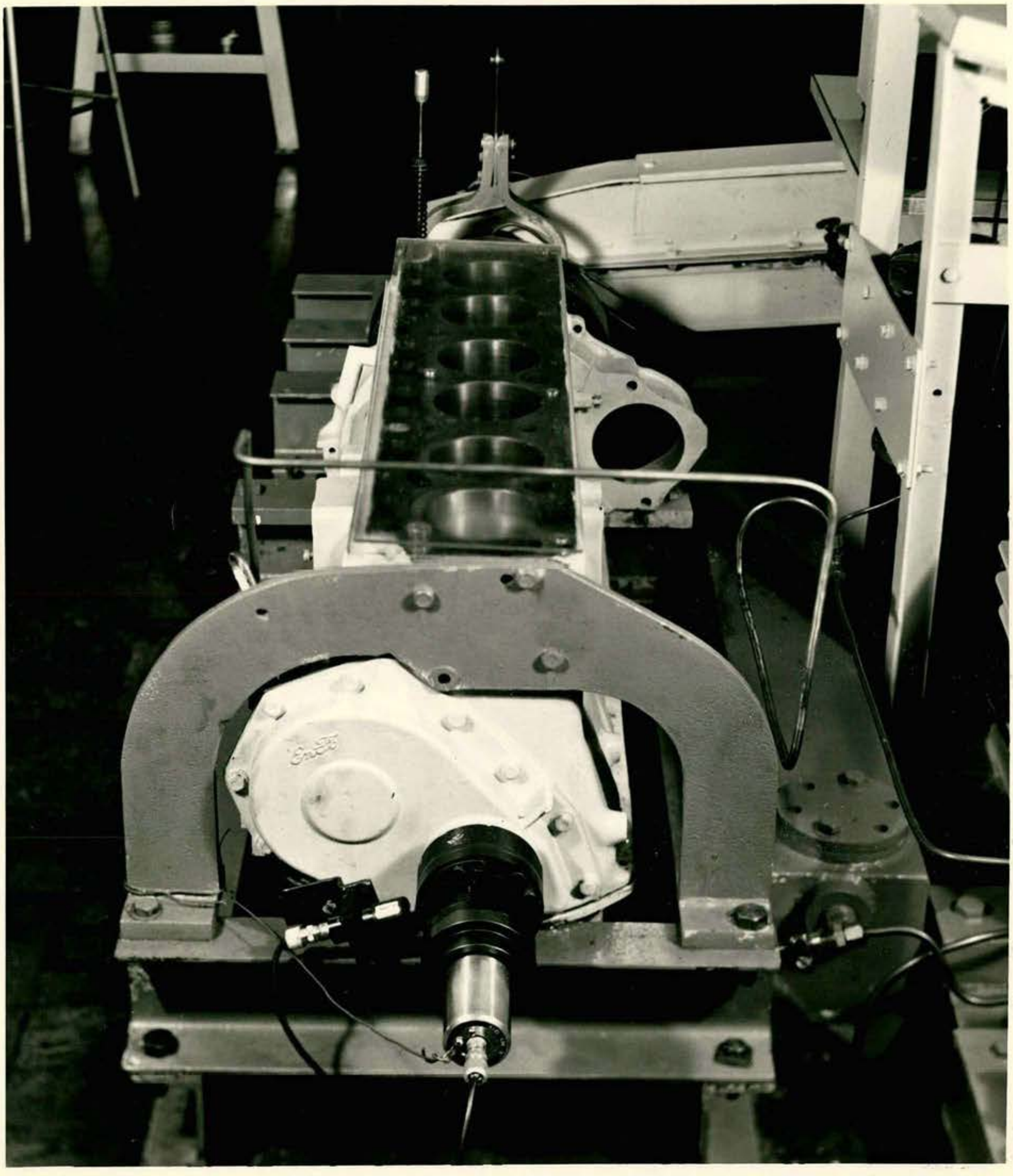
FIG. 18

105

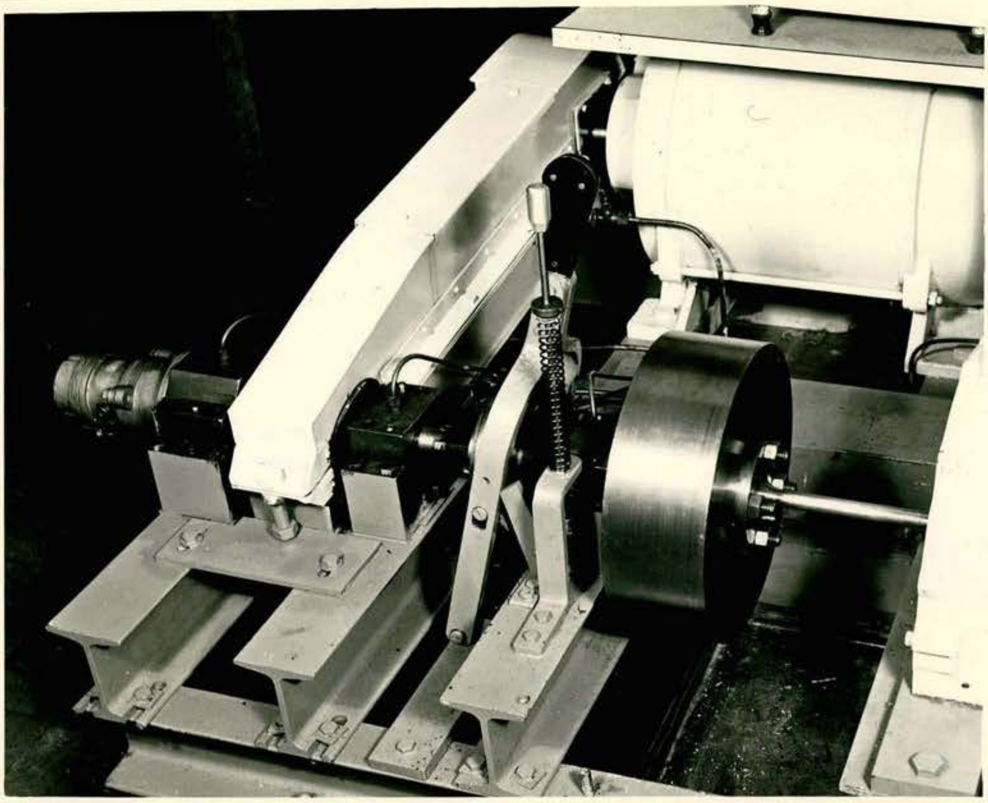

TEMPERATURE, DEGREES FAHRENHEIT
A.S.T.M. STANDARD VISCOSITY-TEMPERATURE CHARTS
FOR LIQUID PETROLEUM PRODUCTS (D 341)
CHART E: KINEMATIC VISCOSITY, LOW-TEMPERATURE RANGE
MOBILOIL SPECIAL.
KINEMATIC VISCOSITY, CENTISTOKES
KINEMATIC VISCOSITY, CENTISTOKES
TEMPERATURE, DEGREES FAHRENHEIT



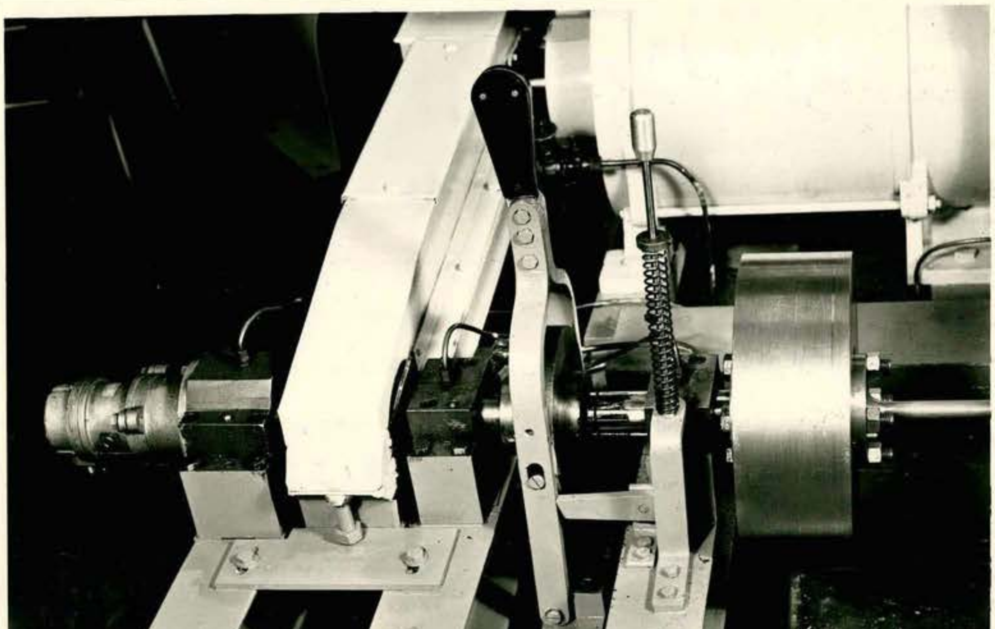
1. General arrangement of the rig with the Ford Zephyr.



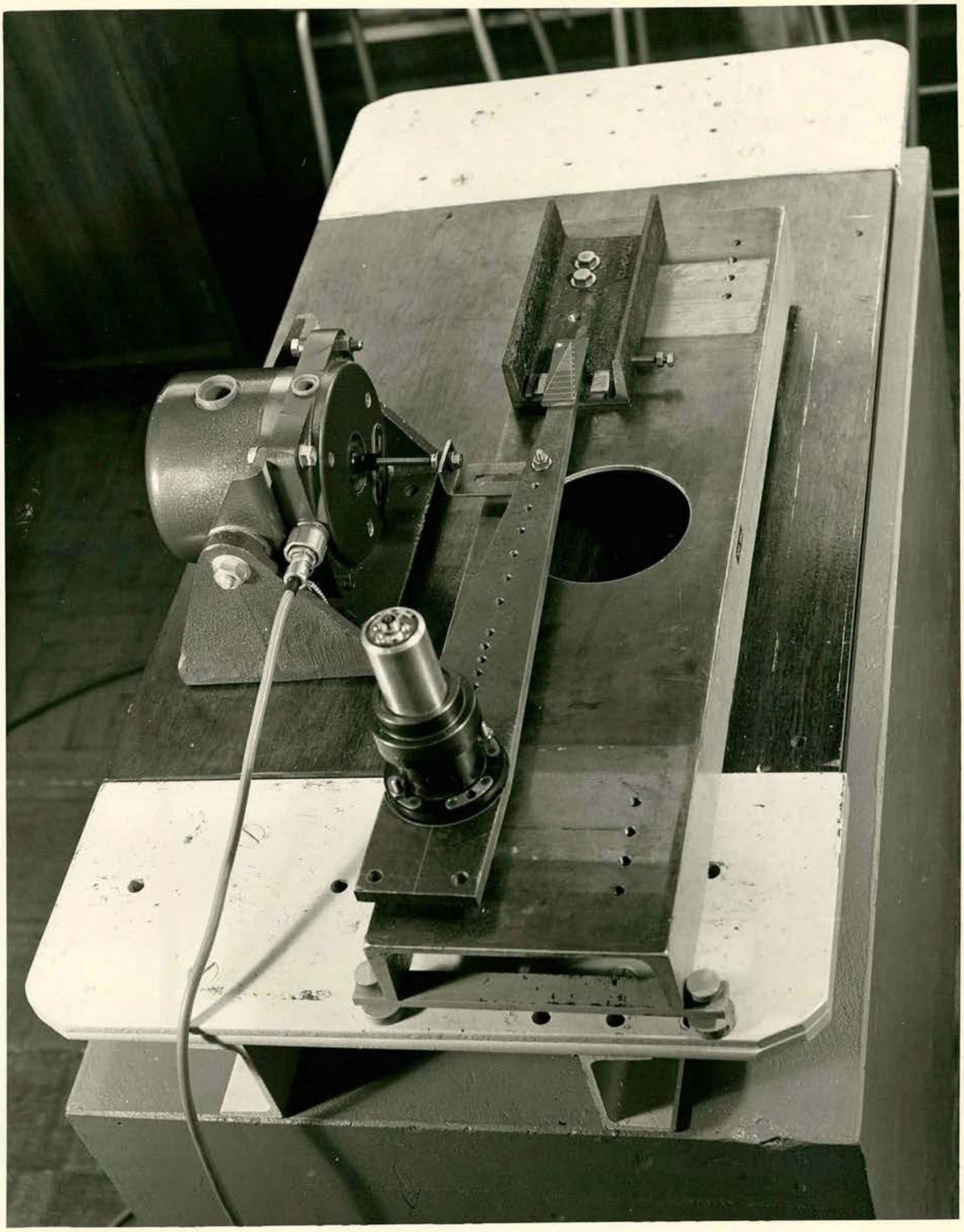
2. Arrangement of the torsional vibration transducer and the revolution marker.



3. The clutch mechanism, engaged.



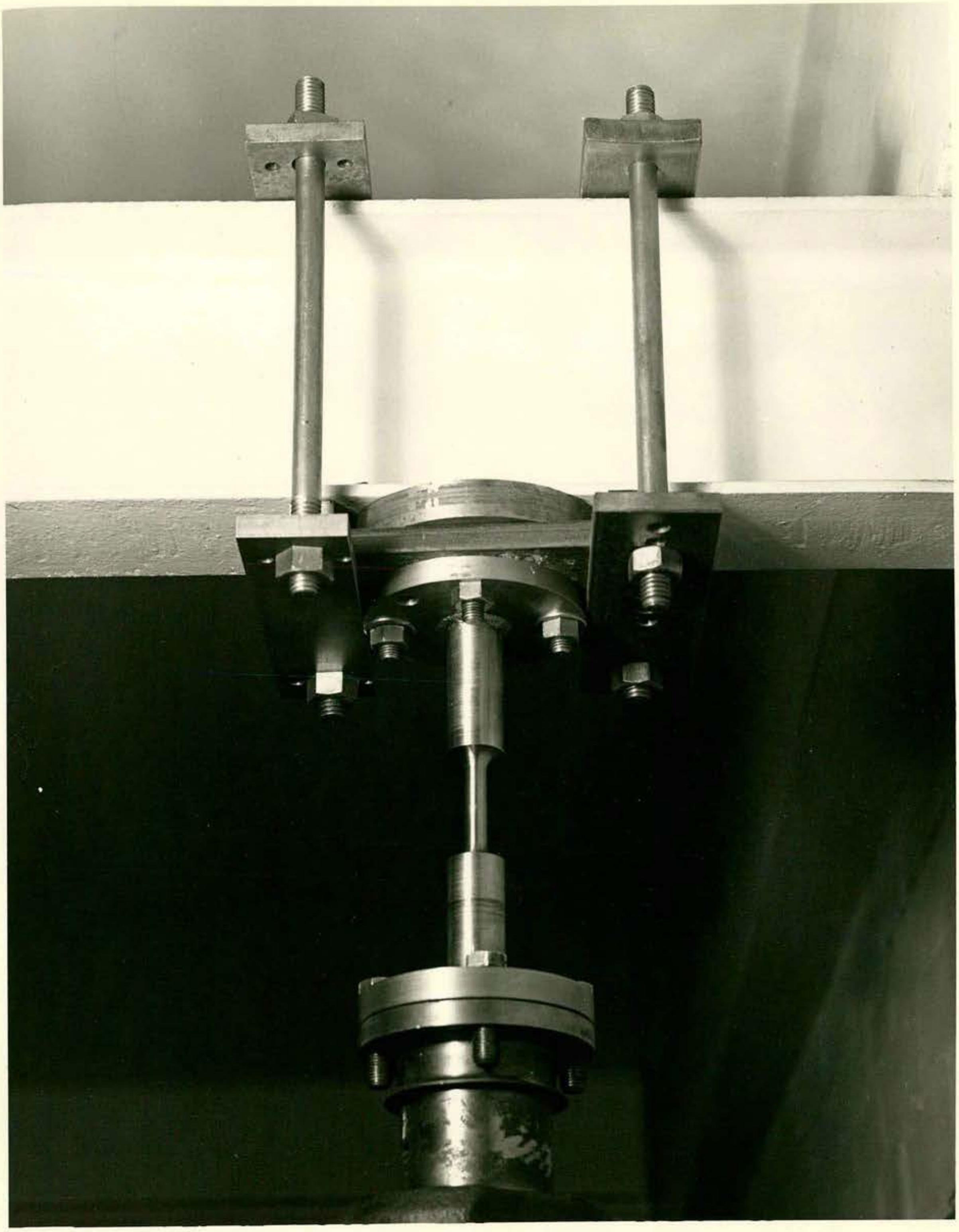
4. The clutch mechanism, withdrawn.



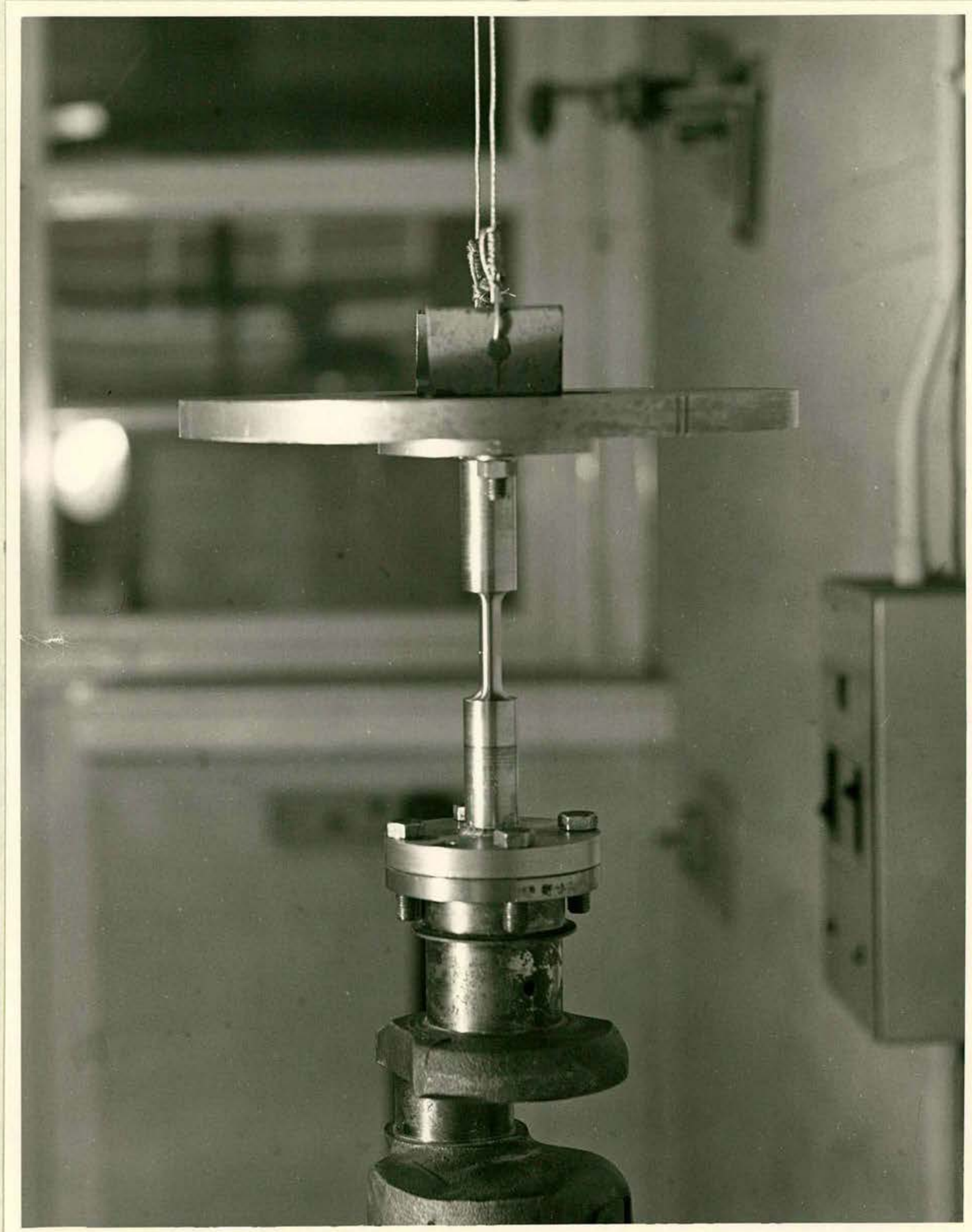
5. Device for calibrating the transducer.

6. Intermediate shaft rigidly mounted for log decrement test.

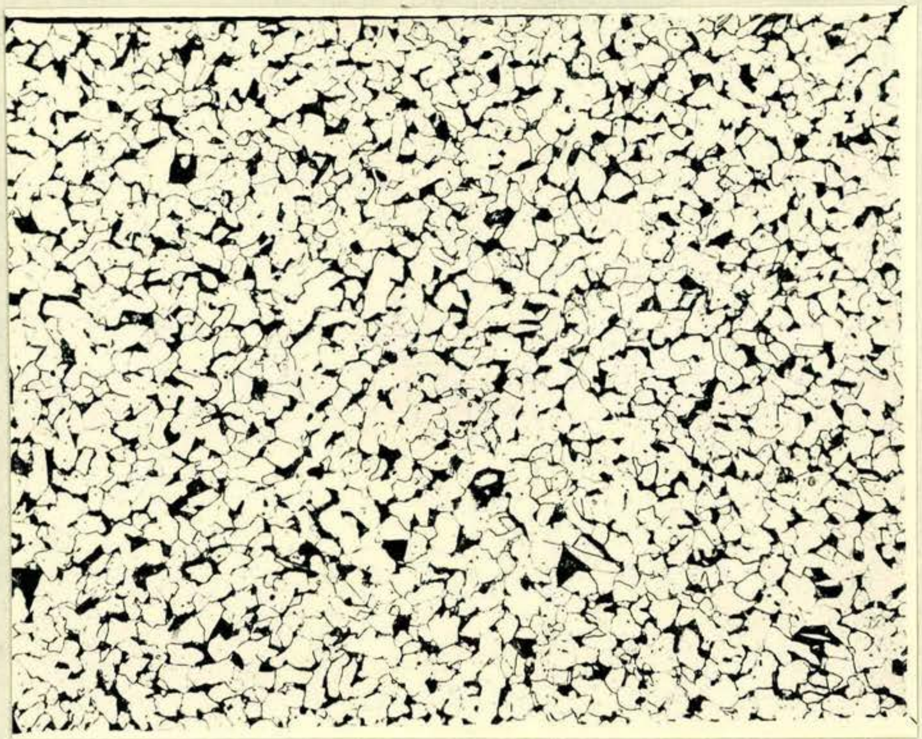
. 7. Intermediate shaft on bifilar suspension for log decrement test.



, 8. The waisted shaft for log decrement tests.



9. The waisted shaft on bifilar suspension.



Photograph 10.

Microstructure of the intermediate shaft
steel X 100 etched in nital.
(white ferrite, black pearlite)

a typical 0.2% C M.S. normalized

ACKNOWLEDGEMENTS

The author wishes to acknowledge with grateful thanks the assistance of The Ford Motor Co. who donated the Zephyr engine for this research, and the encouragement given by Dr. W. A. Pullman, Head of the Mechanical Engineering Department at Rugby College of Engineering Technology, during the early part of the research and by Dr. D. C. Chandler, Head of the Department during the latter part.

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